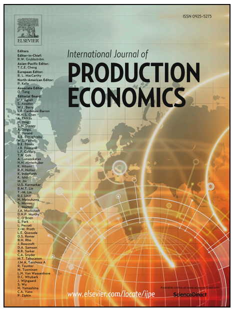
Accepted Manuscript



Subcontracting strategies with production and maintenance policies for a manufacturing system subject to progressive deterioration

H. Éctor Rivera-Gómez, Ali Gharbi, Jean-Pierre Kenné, Oscar Montaño-Arango, Eva Selene Hernández-Gress

PII: S0925-5273(18)30121-X

DOI: 10.1016/j.ijpe.2018.03.004

Reference: PROECO 6974

To appear in: *International Journal of Production Economics*

Received Date: 29 June 2017

Revised Date: 6 January 2018

Accepted Date: 7 March 2018

Please cite this article as: Rivera-Gómez, H.É., Gharbi, A., Kenné, J.-P., Montaño-Arango, O., Hernández-Gress, E.S., Subcontracting strategies with production and maintenance policies for a manufacturing system subject to progressive deterioration, *International Journal of Production Economics* (2018), doi: 10.1016/j.ijpe.2018.03.004.

Subcontracting strategies with production and maintenance policies for a manufacturing system subject to progressive deterioration

HÉCTOR RIVERA-GÓMEZ[1], ALI GHARBI[2], JEAN-PIERRE KENNÉ[3],
OSCAR MONTAÑO-ARANGO[1], EVA SELENE HERNÁNDEZ-GRESS[1]

[1] Academic Area of Engineering, Autonomous University of Hidalgo,
Pachuca-Tulancingo Road km. 4.5, City of Knowledge, Mineral de la Reforma,
Hgo, Mexico, 42184, hriver06@hotmail.com, oscarma11@hotmail.com,
evaselenehg@hotmail.com

[2] Automated Production Engineering Department, École de technologie supérieure,
Production System Design and Control Laboratory, Université du Québec,
1100 Notre-Dame Street West, Montreal, QC, Canada, H3C 1K3
ali.gharbi@etsmtl.ca

[3] Mechanical Engineering Department, École de technologie supérieure,
Laboratory of Integrated Production Technologies, Université du Québec
1100 Notre-Dame Street West, Montreal, QC, Canada, H3C 1K3
jean-pierre.kenne@etsmtl.ca

Abstract

This paper investigates the case of a manufacturing system subject to several progressive deterioration processes through an integrated model that seeks to determine the subcontracting, production and maintenance strategies employed simultaneously. The model is based on the premise that such deterioration affects several performance indices of the machine, centered mainly on the quality of the parts produced and on its reliability. When a machine fails, a minimal repair is conducted and preventive maintenance is available to restore the machine to initial conditions. The control policies indicate the production and preventive maintenance rates, as well as the amount of subcontracting required as a support measure to satisfy product demand. The main objective of the model is to minimize the discounted overall cost, which comprises production, subcontracting, inventory, backlog, preventive maintenance, defectives and repair costs. Hence, we develop a stochastic optimal control model, and numerical methods are used to solve optimality conditions in order to define the structure of the control policies. A simulation-based approach comprising statistical analysis and optimization techniques is applied to determine the optimal parameters associated with the structure of the control policies. The results obtained highlight the strong relations between production, maintenance, deterioration, reliability and quality, which justify the development of an integrated model. Through a numerical example and an extensive sensitivity analysis, we validate our approach.

Keywords: Production planning, Subcontracting, Preventive maintenance, Quality deterioration, Reliability deterioration, Simulation.

1. Introduction

In today´s economic context, subcontracting is gaining considerably in significance and emphasizes the need for better cooperation, coordination and agility between manufacturing companies in order to satisfy customers in terms of quantity and time. Although subcontracting has generated substantial discussion at a general level, very little research has been done from an operational perspective, accompanied by practical implementations. Furthermore, in the literature, relevant issues such as subcontracting, production and maintenance planning have been treated separately, despite their evident interaction. The traditional approach to dissociate decisions has become limited, and no longer satisfies the industrial needs to guarantee maximum availability of production systems, high standards of quality and customer satisfaction. Furthermore, modern production systems use their equipment at high rates without taking into account the fact that production units may degrade rapidly. As a result, efficient management and decision making are therefore imperative for improving the performance of industries. The primary objective of this research is to develop an integrated model that permits the determination of subcontracting activities within an integrated production and maintenance planning approach in a deterioration context. In the next paragraphs, we survey the literature on recent aspects of deterioration, the integration of production/maintenance and subcontracting.

In real industrial environments, components are usually unreliable and maintenance decisions must be integrated at a tactical level. In this paper the focus is on preventive maintenance strategies, and several studies have examined joint production and preventive maintenance planning; that is the case for example of the work of Chelbi et al. (2004), which considered a repairable production unit subject to random failures, and that supplies a subsequent assembly line. In their model, preventive maintenance is regularly performed at given instants. Kouedeu et al. (2015) proposed a hierarchical decision making approach based on the determination of the mean time to failure and the joint optimization of production, preventive and corrective maintenance policies. Along the same lines, Bouslah et al. (2016) investigated the case of production and preventive maintenance strategies for an unreliable production system used to design and optimize a continuous sampling plan subject to quality constraints. As can be noticed preventive maintenance have received considerable attention in the literature and various models have been developed to quantity the effects of investment in preventive maintenance such as the paper of Lee (2005). In this paper the author developed a cost/benefit model for supporting investment strategies in inventory and preventive maintenance for an imperfect production system, where the investment in preventive maintenance reduces defects. Further, this model captures the return on the investment in preventive maintenance and inventory in one common metric. Later, Lee (2008) developed an analytical model for investments in preventive maintenance that reduce the proportion of defectives and also they increase the reliability of a multi-level assembly system. With this model the decision makers can measure the impact of improvement projects and also they can predict the return of an investment in such projects based on the assessment of tangible variables. In manufacturing industries, deterioration is a common industrial phenomenon that can influence the relation between production and preventive maintenance. However, there are scant investigations of production, preventive maintenance and quality policies for deteriorating systems. Furthermore, at taking into account the fact that using the production unit at high rates may degrade it progressively, thereby leading to the lack of capacity and additional costs due to deterioration, may encourage companies to use subcontracting. Therefore, an integrated model, which considers these issues, is needed. In the next paragraph, we review the deteriorating systems domain.

Deterioration is consequential in many real life systems because of its importance in operations management, this was noted by Hajej et al. (2011), who derived optimal production and maintenance schedules for a manufacturing system satisfying random demand considering the deterioration of the machine. Colledani and Tolio (2012) analyzed the production rate of conforming parts of a multi-stage manufacturing system with progressively deteriorating machines and preventive maintenance. In Khatab et al. (2014), a production system subject to stochastic deterioration is considered. In their study, to asses such degradation, the proposed maintenance model considers both corrective and preventive maintenance, a context in which after a given number of maintenance actions, the system is preventively replaced by a new one. Kouki et al. (2014) worked on a mathematical model of a single machine subject to random failure and producing progressive deteriorated products, with preventive maintenance actions applied in order to reduce the expected total cost. In Chouiki et al. (2014), a condition-based maintenance model is proposed for a single-unit production system. Their system is subject to random deterioration, and preventive replacement occurs whenever the level of the system degradation reaches a specific threshold level. The potential effects of machine deterioration have been also integrated in predictive maintenance models such as the paper of Lu et al. (2007) where they determined optimal production quantity and appropriate time to perform maintenance based on the prediction of the future deterioration system condition. However, as prediction is one of the most important parts in predictive maintenance, it also represents its biggest drawback since the employed forecasting techniques are effective for short-term predictions, and their accuracy decreases with the increase of the prediction horizon as noted by Wen et al. (2016). Additionally, most of the studies considered that the manufacturing systems experiences increasing deterioration due to usage and wear. Frequently this deterioration results in limited production capacity, increasing nonconforming and maintenance cost. Nevertheless, we conjecture that much research remains to be done due to the present lack of effective decision methods. In the context of limited production capacity, subcontracting represents an attractive option for the decision maker.

The relation between subcontracting and production strategies has spurred significant research in recent years, since it is an effective strategy for ensuring an improvement of production flexibility, avoiding resource shortages, reducing costs, as illustrated, for instance, in the work of Gharbi et al. (2011). They addressed the production control problem of an adjustable capacity-manufacturing cell, where due to availability fluctuations, the central machine may fall short of meeting long-term demand. In their model, a standby machine is thus called upon support if the finished product inventory level drops below a specific threshold level. The work of Hajej et al. (2014) contributed to the study of an industrial case where the use of a subcontractor takes over any remaining production in order to satisfy customer demand. In their model, preventive maintenance is used to reduce the failure frequency. Assid et al. (2015) proposed the joint optimization of production and subcontracting of multiple production facilities with different capacities, where subcontracting compensates for insufficient production capacity. Additionally, Rivera-Gómez et al. (2016) derived a control policy which determined the production and overhaul rates and the rate at which subcontractor products are requested. The authors assumed that subcontracting options are available at a higher cost to supplement the limited production capacity. Assuming that the failure rate deteriorates, and preventive maintenance can be conducted to mitigate the effect of deteriorations, a model was presented by Haoues et al. (2016) to coordinate in-house production, subcontracting and maintenance plans. In the area of subcontracting, relatively few studies have considered the effects of deterioration, mainly focusing on its influence on only one system parameter (i.e. failure rate). In contrast, we analyze the case where such deterioration may be amenable to severely affect several system parameters (for instance, the failure rate of the machine and the quality of the parts produced) due to the production environment, accumulated wear, usage, etc.

The motivations of this paper are directed towards the extension of common assumptions presented in the literature, noting that most studies consider the perspective of the subcontractor or of the producer, primarily, and optimize only one party. Unfortunately, this does not necessarily lead to an optimal situation for the whole system. In this paper, we study complex industrial features that have not been covered in an integrated fashion in the literature. We will consider the following features simultaneously: i) machines characterized by multiple progressive degradations and wears, ii) reduction of the production capacity due to the machine´s progressive deteriorations, iii) optimization of the time when preventive maintenance is performed,

iv) optimization of the quantity of products to produce in-house and quantity of products to be supplied by subcontracting, and v) the determination of the appropriate time when subcontractor should be called. The paper is intended to assist managers with an integrated policy that indicates how to balance the trade-off between in-house production, subcontracting and maintenance. The proposed model seeks to minimize the total incurred cost, composed of the inventory, backlog, preventive maintenance, production, subcontracting, repair and defectives costs. We use a stochastic optimal control model formulation, whose optimality conditions are resolved numerically to determine the structure of the obtained control policies. A simulation-based approach combined with design of experiments (DOE), optimization techniques and simulation modeling is adopted to determine the optimal values of the control parameters. An extensive sensitivity analysis is conducted to provide a better insight into the production system behavior. In addition, a manufacturing system composed of several parallel machines is also studied to illustrate the flexibility of the proposed approach.

The remainder of this paper is organized as follow: The usefulness and the industrial impact of the paper are discussed in Section 2. In Section 3, we present the problem formulation and notations. The structure of the joint control policy is detailed in Section 4. In Section 5, we present the simulation-based approach adopted. Section 6 introduces a numerical example. In Section 7, we illustrate the policy implementation. Section 8 presents an extensive sensitivity analysis. A comparative study between our integrated production, subcontracting and preventive maintenance control strategy and a policy that disregards subcontracting issues is detailed in Section 9. Finally, Section 10 concludes the paper.

2. Usefulness and industrial impact

The phenomenon of deterioration is common in automobile, aircraft, machine tools and paper manufacturing plants, typically in systems comprised of a large number of components which stochastically deteriorate over time (Kouedeu et al. (2015)). Additionally, the cumulative use of the manufacturing unit may accelerate the machine degradation, and therefore increase the risk of failures, defectives, delays, etc. Consequently, the number of failures and defective products, increases. If production is carried on with such a system, it degrades further (i.e. there are more failures and more defectives), which may further limit the in-house production capacity. Once the manufacturing system is unable to satisfy the customer demand, additional costs due to deterioration will encourage companies to adopt subcontracting in order to improve the limited in-house production capacity, as has been noted in several sectors, such as the textile, retail channel, pharmaceutical and semiconductor industries. Therefore, given this context, we developed a stochastic dynamic programming model, which can be used to analyze manufacturing systems facing progressive reduced production capacity caused by deterioration.

In many production systems, subcontracting is adopted at a higher cost to enhance limited in-house production capacity. If properly handled, subcontracting can shorten lead times, reduce total costs, and render organizations more flexible, as well; it is commonly used as a tool to improve overall planning effectiveness. Furthermore, it can provide a competitive advantage to companies and maintain core competition (Haoues et al. (2016) and Dror et al. (2009)).

3. Problem formulation

In this section, we detail the notations used in this article, and present a general description of the manufacturing system under study. Additionally we formulate the optimal control problem.

3.1 Notations

The following definitions are used throughout the paper:

$x(t)$	Inventory level at time *t*
$a(t)$	Age of the machine at time *t*
d	Constant demand rate of products, (products/day)
$\xi(t)$	Stochastic process $\{\xi(t), t \geq 0\}$
$\lambda_{\alpha\alpha'}(\cdot)$	Transition rate from mode α to mode α'
π_i	Limiting probability at mode *i*
$u(\cdot)$	Production rate of the manufacturing system, (products/day)
$\bar{u}$	Maximum production rate of the producer, (products/day)
$\theta(\cdot)$	Preventive maintenance rate
$\bar{\theta}$	Maximum preventive maintenance rate
$\underline{\theta}$	Minimum preventive maintenance rate
$v(\cdot)$	Subcontracting rate, (products/day)
$\bar{v}$	Maximum subcontracting rate, (products/ day)
τ	Fraction of demand satisfied by subcontracting
$\beta(\cdot)$	Rate of defectives
c^{+}	Inventory holding cost, ($/products/day)
c^{-}	Backlog cost, ($/products/ day)
c_r	Repair cost ($)

c_{pm}	Preventive maintenance cost (\$)
c_d	Cost of defectives (\$/product)
C_{pro}	Cost of in-house production (\$/product)
C_{sub}	Cost of subcontracting (\$/product)
$g(\cdot)$	Instantaneous cost function
ρ	Discount rate
$J(\cdot)$	Expected discounted cost function
$V(\cdot)$	Value function
C^*	Minimum total cost (\$/day)

For the rest of the paper, the time unit is the day and the cost unit is the dollar ($).

3.2 Modelling assumptions

The model developed in this paper is based on the following assumptions:

1) The machine deteriorates progressively with its operating age.
2) The effects of the deterioration process are observed in the increase of the failure rate and the rate of defectives, limiting its production capacity.
3) At failure a minimal repair is conducted, leaving the machine to as-bad-as-old conditions.
4) When preventive maintenance is conducted it implies a perfect maintenance that restores the machine to initial conditions.
5) Subcontracting is always available and serves to mitigate the reduction of the production capacity of the manufacturing system.
6) Subcontracting supplies products free of defectives.
7) The production rate, subcontracting and the preventive maintenance rates are controlled.

3.3 Description of the manufacturing system

The manufacturing system under study refers the case of an unreliable machine which produces one part type. Figure 1 presents a graphical representation of such a system. The manufacturing system is unreliable since it is subject to random breakdowns and repairs. The main assumption of the model is that such machine is subject to a deterioration process with multiple effects, which are directly related to the quality of the parts produced, as well as to its reliability. To mitigate the effects of deterioration, preventive maintenance can be conducted. In fact, in our case, maintenance decisions are based on real deterioration condition instead of predictions; with this, we have the advantage of a more accurate deterioration model. In this context of progressive deterioration, the manufacturing system will eventually no longer be capable of satisfying demand with flawless products. Therefore, subcontracting becomes an attractive alternative for this manufacturing system characterized by a limited production capacity to fulfill product demand. The objective of the model is to optimize the subcontracting contribution, and simultaneously determine production and preventive maintenance strategies to minimize the total incurred cost, which includes the inventory, backlog, production, subcontracting, preventive maintenance, repair and defectives cost of units in the case where the deterioration process of the machine has a twofold effect, observed mainly on the quality of the parts produced and also on its reliability.

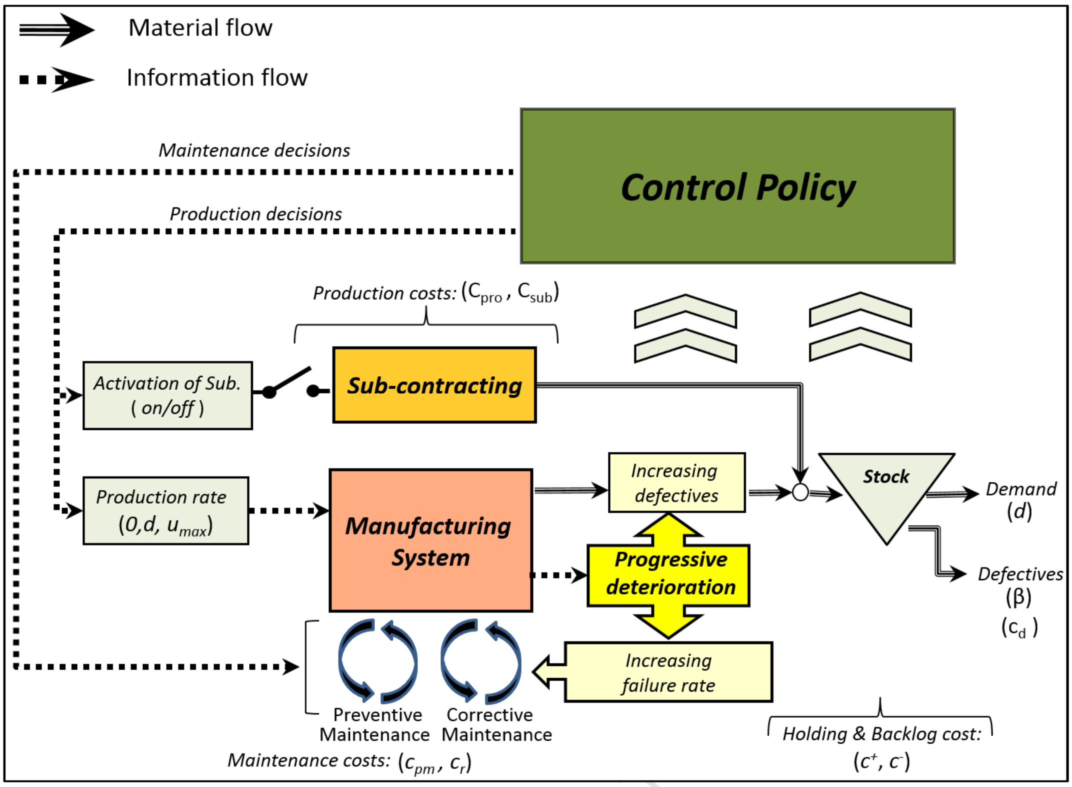


Figure 1: Block diagram of manufacturing system under analysis

3.4 Control model formulation

We will now formulate the control problem, based on the manufacturing system of Figure 1. The machine is subject to random events (failures, repairs, and preventive maintenance), and also to a deterioration process. It is operational when $\{\xi(t) = 1\}$, and unavailable when under repair $\{\xi(t) = 2\}$ or under preventive maintenance $\{\xi(t) = 3\}$. Hence, we define three modes, described by the stochastic process $\{\xi(t), t \geq 0\}$, with values in $\Omega = \{1,2,3\}$. On an infinite horizon, the machine can stay randomly in each of the three modes according to the state transition diagram presented in appendix A. When the machine fails (mode 2), it is immediately repaired at a constant rate λ_{21}. Repair involves a minimal maintenance that leaves the machine in an as-bad-as-old (ABAO) condition; the history of the machine is thus needed in modeling this type of maintenance, which therefore means that a semi-Markov model is required. In our formulation, we use the age of the machine $a(t)$ as a state variable of the system to take into account this history. Meanwhile, preventive maintenance (mode 3) implies a perfect maintenance that restores the machine to an as-good-as-new (AGAN) condition at a constant rate λ_{31}.

Let d be the product demand, and $u(t)$ the production rate at time t. At any instant of time, the production rate must satisfy the capacity constraint: $0 \leq u(\cdot) \leq \bar{u}$, where $\bar{u}$ is the maximum production rate of the machine. Let $v(t)$ the subcontracting rate, with value in $\{0, \tau \cdot d, d\}$ such that $(0 < \tau < 1)$.

In our model, the first issue to be handled with respect to deterioration is to specify that it has multiple effects on the machine, and that these effects are reflected mainly on the part quality and machine reliability. First, we conjecture that deterioration progressively increases the rate of defectives; to model this condition, we therefore propose that the dynamics of the inventory/backlog of products $x(t)$ evolves according to the following differential equation:

$$\frac{dx(t)}{dt} = u(t) \cdot (1 - \beta(a)) + v(t) - d, \qquad x(0) = x_o \tag{1}$$

where x_o refers to the initial inventory level and $\beta(a)$ represents the rate of defectives as a function of age. We define the age of the machine at time t as a function of its production rate since the last restart, as follows:

$$\frac{da(t)}{dt} = k_1 \cdot u(t), \qquad a(T)=0 \tag{2}$$

where the parameter k_1 denotes a positive constant and T is the last restart time of the machine following preventive maintenance. The deterioration process is characterized by a decreasing yield, originating tool wear or specific machine aging. Normally, phasetype distributions can be used to model such deteriorations, and this phenomenon can be discretized, as presented in Figure 2a. An alternative means to model machine degradation involves using an increasing function (*IF*), as presented in the dotted line in Figure 2a, which provides a technical advantage of maintaining a tractable state space. In particular, we use an *IF* to model different quality yields, as suggested by Kim and Gershwin (2008) and Colledani and Gershwin (2013). Therefore, we have:

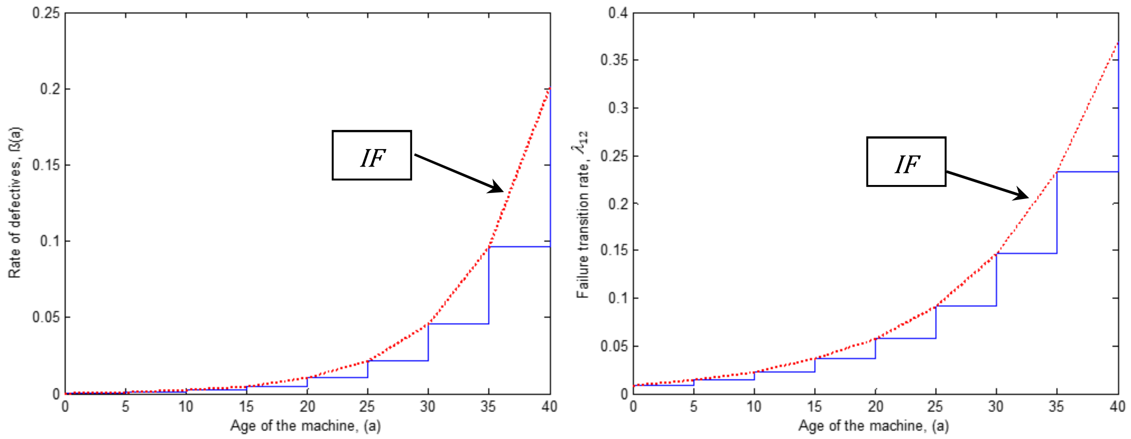
$$\beta(a) = (r_d)^{a-1} \cdot \beta_o \tag{3}$$

where β_o is the initial value of the rate of defectives, r_d represents the common ratio used to adjust the expression to a set of data. As an illustration, we present the trend for the rate of defectives in Figure 2a; for real systems r_d and β_o could be obtained by analyzing historical data.

In establishing the second effect of deterioration, which is mainly observed at the level of reliability of the machine, we assume that as the machine deteriorates, it fails more frequently, which in effect means that its failure rate increases. We use a second *IF* to model this condition, as noted by Love et al. (2000) and Dehayem-Nodem et al. (2011a), since deterioration can be the result of the effect of several factors, such as usage, corrosion, environment, etc. Hence, the failure rate is given by:

$$\lambda_{12}(a) = (r_q)^{a-1} \cdot q_o \tag{4}$$

where the parameter r_q is used to adjust the trend of the failure rate and q_o is the value of the transition λ_{12} at initial conditions. In Figure 2b, we present a realization of the failure rate. Equation (4) states that the failure rate of the machine depends on its age $a(t)$, thus leading to the case of a non-homogeneous semi-Markov process which serves to model increasing failures. Other transitions $\lambda_{\alpha\alpha'}(\cdot)$ from mode α to mode α' are illustrated in appendix A. For real systems, historical data is used to determine the appropriate value for r_q and q_o. Equations such as (3) and (4) have been successfully applied to model time intervals between successive failures of industrial equipment, as reported in Lam (2004).



a) Rate of defectives (with $r_a = 1.16$ and $\beta_o = 0.01$)
b) Rate of failures (with $r_q = 1.097$ and $q_o = 0.01$)

Figure 2: Increasing Function

The decision variables of the model are the production rate of the machine $u(\cdot)$, the subcontracting rate $v(\cdot)$ and the preventive maintenance rate $\theta(\cdot)$. We assume that the following constraint holds for the preventive maintenance rate:

$$\underline{\theta} \leq \theta(\cdot) \leq \bar{\theta} \tag{5}$$

Where $\underline{\theta}$ and $\bar{\theta}$ are given constants. The transition rate from the operational mode to the preventive maintenance mode λ_{13} is a control variable denoted $\theta(\cdot)$ and its physical interpretation is that the reciprocal $1/\theta(\cdot)$, represents the expected delay between a call for the technician and his arrival. Thus, when $\theta(\cdot)$ is set to its maximal value $\bar{\theta}$, preventive maintenance is conducted almost immediately after a short delay, and when $\theta(\cdot)$ is set to its minimal value $\underline{\theta}$, the delay for preventive maintenance takes so much time that this maintenance is not conducted. With the control of transition rates

(λ_{13}= $\theta(\cdot)$), their dependence on state variables ($\lambda_{12}(a)$) and the fact that reparations are minimal (ABAO), the resulting process is said to be a non-homogeneous controlled transition rates semi-Markovian process.

Referring to the maximum principle used in dynamic programming, which is based on the initial conditions (e.g., for a given age), we assessed the feasibility of the system in the absence of preventive maintenance (i.e., $\theta(\cdot) = \underline{\theta}$). The feasibility condition is given by the expression $\bar{u}(\cdot) \cdot [1 - \beta(a)] \cdot \boldsymbol{\pi}_1(a) \geq d$, where $\pi_1(a)$ is used here to approximate the limiting probability at operational mode of the production system for a given age of the machine. In order to study the evolution of the feasibility condition as a function of age, the limiting probabilities for modes 1, 2 and 3 could be calculated for each age of the machine as follows:

$$\boldsymbol{\pi}(\cdot)Q(\cdot) = 0 \text{ and } \sum_{i=1}^{3} \pi_i = 1 \tag{6}$$

where $\boldsymbol{\pi}(\cdot) = (\pi_1(\cdot), \pi_2(\cdot), \pi_3(\cdot))$ is the vector of limiting probabilities. The resolution of Equation (6) gives $\pi_1(a) = 1/(1 + \lambda_{12}(a)/\lambda_{21} + \underline{\theta}/\lambda_{31})$ describing the limiting probability of the operational mode at a given age of the machine.

However, due to the deterioration effects, the failure rate and the rate of defectives increases continuously, limiting then the production capacity of the unit. Thus, the decision-maker must ensure that the manufacturing system is able to satisfy the long-term product demand in cases of extreme deterioration. (i.e. high values of $\lambda_{12}(a)$ and $\beta(a)$). In this case at considering the participation of subcontracting, the whole capacity constraint of the system is given by:

$$\bar{u}(\cdot) \cdot [1 - \beta(a)] \cdot \boldsymbol{\pi}_1(a) + \bar{v} \geq d \tag{7}$$

where $\bar{v}$ is the maximum allowable subcontracting rate. Equation (7) indicates if the contribution of the production system and subcontracting will be able to fulfill product demand at high levels of deterioration. Additionally, given the effects of deterioration, it is evident the need of a preventive maintenance plan to restore the machine and mitigate the impact of such deterioration process.

The set of admissible decisions $\Gamma(\alpha)$, including the decision variables (u, v, θ) is:

$$\Gamma(\alpha) = \tag{8}$$

$$\left\{\big(u(\cdot), v(\cdot), \theta(\cdot)\big) \in \mathbb{R}^3, \quad 0 \le u(\cdot) \le \bar{u}, \qquad 0 \le v(\cdot) \le \bar{v}, \qquad \underline{\theta} \le \theta(\cdot) \le \overline{\theta}, \right\}$$

The performance measure of the system is denoted by the instantaneous cost function of the model at mode $\alpha \in \Omega$, which can be written as:

$$g(\alpha, x, a, u, v, \theta) = \tag{9}$$

$$c^+x^+(t) + c^-x^-(t) + C_{pro} \cdot u(t) + C_{sub} \cdot v(t) + c_d \cdot \beta(a) \cdot u(t) + c^\alpha, \quad \forall\, \alpha \in \Omega$$

with:

$$x^+(t) = max\big(0, x(t)\big)$$

$$x^-(t) = max(-x(t), 0)$$

$$c^\alpha = C_r \cdot Ind\{\xi(t) = 2\} + C_{pm} \cdot Ind\{\xi(t) = 3\}$$

$$Ind\{\xi(t) = \alpha\} = \begin{cases} 1 & if\ \xi(t) = \alpha \\ 0 & otherwise \end{cases}$$

where c^+ and c^- are the inventory and backlog cost, respectively; C_{pro} is the production cost of the manufacturing system; C_{sub} is the subcontracting cost, and c_d refers to the cost due to the additional inspection and disposal of defective parts; C_r is the repair cost and C_{pm} defines the preventive maintenance cost. Further, it is assumed that the cost of subcontracting, C_{sub}, is much higher than the in-house production cost, C_{pro}, thus $0 < C_{pro} \ll C_{sub}$. Our objective is to find in Γ(α) the control policies (u^*, v^*, θ^*) which minimizes, for each initial condition (α, x, a) the following discounted cost

$$J(\alpha, x, a) = E\left[\int_0^{\infty} e^{-\rho t} g(\cdot) dt \mid \quad \xi(0) = \alpha, x(0) = x, a(0) = a\right] \tag{10}$$

where ρ is the discount rate. The value function of the problem is defined by:

$$V(\alpha, x, a) = \inf_{(u,v,\theta)\in\Gamma(\alpha)} J(\alpha, x, a) \tag{11}$$

The optimal control problem as formulated in this article belongs to a class of problems widely studied in the literature (i.e. Boukas and Haurie (1990), Dehayem-Nodem et al., (2011b), Ouaret et al. (2018) and references cited therein). The properties of the value function $V(\alpha, x, a)$ given by equation (11) and how to obtain the optimality conditions, described by Hamilton–Jacobi–Bellman (HJB) equations can be found specifically in Gershwin (2002) and in Dehayem-Nodem et al. (2011b). Applying the maximum principle and adapting the HJB equations presented in Boukas and Haurie (1990) and in Dehayem-Nodem et al. (2011b), we obtained the following optimality conditions:

$$\rho V(\alpha, x, a) = \tag{12}$$

$$\min_{(u,v,\theta)\in\Gamma(\alpha)}\left[g(\cdot) + \frac{\partial V}{\partial x}(\cdot)(u \cdot [1-\beta(a)] + v - d) + \frac{\partial V}{\partial a}(\cdot)(k_1 \cdot u) + \sum_{\alpha\in\Omega} \lambda_{\alpha\alpha'}(\cdot) V\big(\alpha, x, \varphi(\xi, a)\big)(\alpha)\right]$$

Where $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial \mathrm{a}}$ are the partial derivatives of the value function. Equations (12) is complemented with the following reset function:

$$\varphi(\xi, a) = \begin{cases} 0 & if \;\; \xi(\sigma^+) = 1 \quad and \;\; \xi(\sigma^-) = 3 \\ a(\sigma^-) & otherwise \end{cases} \tag{13}$$

Where σ is the first jump time of ξ(t). Equation (13) allows us to model the benefit of preventive maintenance, which is considered as a perfect maintenance that mitigates the effects of the deterioration process, then it restores the age of the machine to AGAN conditions. Also Equation (13) indicates that the minimal repair does not mitigate the deterioration process, since the age of the machine remains in ABAO conditions.

The optimal control policy (u^*, v^*, θ^*) is obtained by solving the HJB equations (12) through the set of admissible decisions $\Gamma(\alpha)$. This policy corresponds to the value function described by Equation (11). However, the HJB equations consist of a set of coupled partial differential equations that cannot be solved analytically. As a result, the use of numerical algorithms to approximate the value function and the corresponding control policy has become a viable alternative to overcome this difficulty of resolution (Boukas and Haurie (1990)). Hence, we adopted a numerical approach to approximate the value function and its associated control policy.

4. Numerical approach and joint optimal control policy

As stated previously, because of the complexity of the set of partial differential equations embedded in the HJB Equations (12), closed-form solutions are not feasible. Hence, we use numerical methods based on the Kushner approach (Kushner and Dupuis (1992)) to determine a solution and define the structure of the optimal control policies. The main idea of this approach is to apply an approximation scheme for the gradient of the value function, where a discrete function $v^h(\alpha, x, a)$ is used to approximate the continuous value function $v(\alpha, x, a)$. Meanwhile, its partial derivatives $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial a}$ are expressed as a function of the discrete value function $v^h(\alpha, x, a)$ and the length of the finite difference interval for the stock and the age of the machine, h_x and h_a, respectively, as follows:

$$\frac{\partial v}{\partial x}(\alpha, x, a) = \begin{cases} \frac{1}{h_x}[v^h((\alpha, x + h_x, a) - v^h((\alpha, x, a)] & if \quad \frac{\partial v}{\partial x} \geq 0 \\ \frac{1}{h_x}[v^h((\alpha, x, a) - v^h((\alpha, x - h_x, a)] & if \quad \frac{\partial v}{\partial x} < 0 \end{cases} \tag{14}$$

and

$$\frac{\partial v}{\partial \mathrm{a}}(\alpha, x, a) = \frac{1}{h_a}[v^h((\alpha, x, a + h_a) - v^h(\alpha, x, a)] \tag{15}$$

At considering Equations (14) and (15), the Kushner´s technique leads to the definition of a discrete counterpart of the HJB Equations (12), which is expressed in terms of the discrete function $v^h(\alpha, x, a)$ with step size h_x and h_a on a discrete grid, as denoted by the next equation:

$$v^h(\alpha, x, a) \ =$$

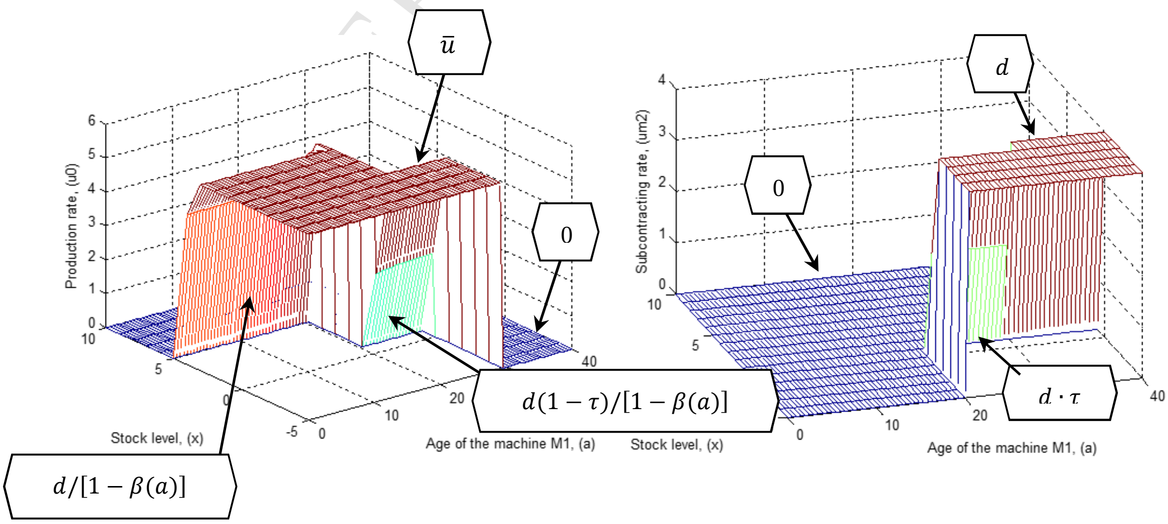
$$\min_{(u,v,\theta)\,\in\,\Gamma(\alpha)} \left[\left(\rho + + \frac{\eta_a}{h_a} + \frac{|\eta_x|}{h_x} + |\lambda_{\alpha\alpha}| \right)^{-1} \left(g(\cdot) + \frac{\eta_a}{ha} v^h(\alpha, x, a + h_a) \right.\right.$$
$$+ \frac{|\eta_x|}{hx} v^h(\alpha, x + h_x, a) Ind\{\eta_x \geq 0\} + \frac{|\eta_x|}{hx} v^h(\alpha, x - h_x, a) Ind\{\eta_x < 0\}$$
$$\left.\left. + \sum_{\alpha \in \Omega} \lambda_{\alpha\alpha'}(\cdot) V(\alpha, x, \varphi(\xi, a)) \right) \right] \quad \forall\, \alpha \in \Omega, \qquad x \in R, \qquad a \in R \tag{16}$$

Where $\eta_a = k_1 \cdot u(t)$ and $\eta_x = u(t) \cdot (1 - \beta(a)) + v(t) - d$. Equation (16) is the discrete counterpart of the HJB Equations (12) that is implemented in the numerical technique. Equation (16) is solved by the policy improvement technique, (for more details see Kenné et al. (2012) and references therein). In Table 1, we present the parameters used in the numerical example.

Parameter:	q_o (λ_{12})	r_q	λ_{31} (1/day)	β_o	r_d
Value:	0.01	1.097	0.3	0.01	1.16
Parameter:	λ_{21} (1/ day)	$\bar{u}$ (product/ day)	$\underline{\theta}$ (1/ day)	$\bar{\theta}$ (1/ day)	d (product/ day)
Value:	1.5	5.5	10^{-6}	20	4
Parameter:	$\bar{v}$ (product/ day)	k_1	h_x	h_a	ρ
Value:	4	0.029	1	0.5	0.9
Parameter:	c+ ($/products/day)	c- ($/products/day)	c_{pro} ($/product)	c_{sub} ($/product)	c_d ($/product)
Value:	1	270	4	40	10
Parameter:	c_r ($/repair)	c_{pm} ($/preventive maintenance)	τ (%)		
Value:	5	10	0.5		

Table 1. Parameters for the numerical example

In what follows, the obtained production policy divides the plan (x, a) into four regions, where the production rate is set to $\bar{u}$, $d/[1-\beta(a)]$, $d(1-\tau)/[1-\beta(a)]$ and 0, respectively, as illustrated in Figure 3a. Moreover, the subcontracting policy divides the plan (x, a) into three regions, where the subcontracting rate is set to 0, $d \cdot \tau$ and d, as shown in Figure 3b.



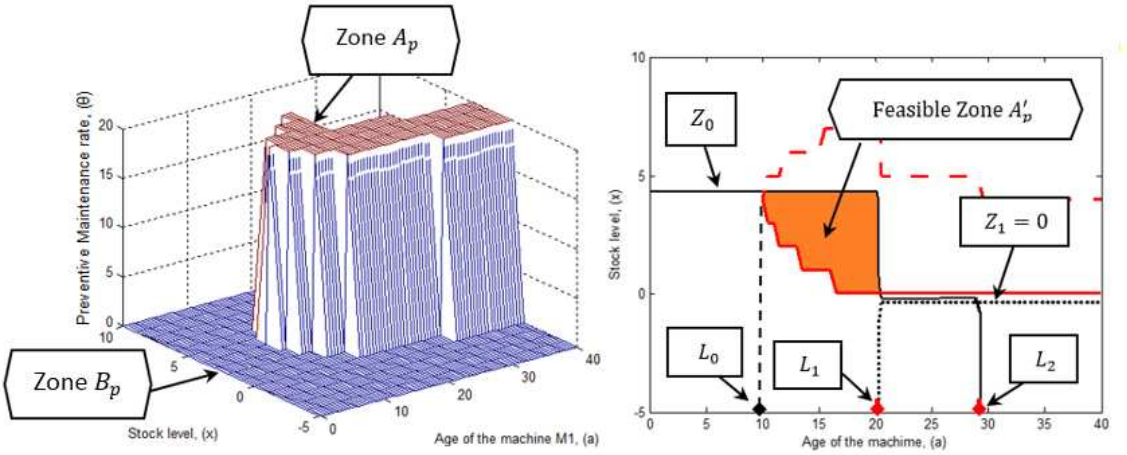
a) Production policy **b) Subcontracting policy**

Figure 3: Control policies

With respect to the ***preventive maintenance policy,*** we identify two zones in Figure 4a described as follows:

- Zone A_p: The recommendation is to perform preventive maintenance activities, then $\theta(\cdot) = \overline{\theta}$.
- Zone B_p: In this zone, preventive maintenance is not recommended, it is more profitable to continue operating the manufacturing system. Thus, $\theta(\cdot) = \underline{\theta}$.

However, in the case of a joint control policy, to characterize the preventive maintenance rate, we must consider simultaneously the production and the preventive maintenance boundaries, as presented in Figure 4b. Since the stock level is limited by the production threshold, and only a part of the preventive maintenance zone A_p is active, this then defines the feasible preventive maintenance zone A'_p.



a) Preventive maintenance policy **b) Production, preventive maintenance and subcontracting trace**

Figure 4: Joint control policy

In view of these results, we notice that production, subcontracting and preventive maintenance policies are highly inter-related. Thus, we must define them simultaneously, and to facilitate this process, we use four age intervals: $a \leq L_0$, $L_0 < a \leq L_1$, $L_1 < a \leq L_2$ and $a > L_2$, where point L_1 indicates the age when $\bar{u} \cdot \boldsymbol{\pi}_1(\mathrm{a}) = [d/(1-\beta(a))]$, after which the capacity constraint of the system (formula (7)) is no longer satisfied, and subcontracting is then required. Note that the subcontractor is not used before the machine age reaches the value of L_1. Point L_2 indicates the age when the machine is so deteriorated that it should be stopped and sent out for preventive maintenance. The obtained results indicate that the ***joint production, subcontracting and preventive maintenance policy*** are thus defined with the set of equations presented in Table 2:

Age-interval	*Production policy*, $u^*(1,x,a)$	*Subcontracting policy*, $v^*(1,x,a)$	*Preventive maintenance*, $\theta^*(1,x,a)$
$a \leq L_0$	$\begin{cases} \bar{u} & if \quad x < Z_0 \\ d/(1-\beta(a)) & if \quad x = Z_0 \\ 0 & if \quad x > Z_0 \end{cases}$	$= 0 \quad \forall x$	$\underline{\theta}$
$L_0 < a \leq L_1$	$\begin{cases} \bar{u} & if \quad x < Z_0 \\ d/(1-\beta(a)) & if \quad x = Z_0 \\ 0 & if \quad x > Z_0 \end{cases}$	$= 0 \quad \forall x$	$\bar{\theta}$ if $x(t) \geq 0$
$L_1 < a \leq L_2$	$\begin{cases} \bar{u} & if \quad x < Z_1 \\ d(1-\tau)/(1-\beta(a)) & if \quad x = Z_1 \\ 0 & if \quad x > Z_1 \end{cases}$	$= d \cdot \tau \quad \forall x$	$\bar{\theta}$ if $x(t) \geq 0$
$a > L_2$	$0 \quad \forall x$	$= \begin{cases} d & if \quad x \leq 0 \\ 0 & otherwise \end{cases}$	$\bar{\theta}$

Table 2. Production, subcontracting and preventive maintenance policies

Where Z_0 represents the stock level that delimits the optimal production threshold, and $Z_1 = 0$, defines the subcontracting trace as observed in Figure 4b. Additionally, we note in Table 2 that in the interval $L_1 < a \leq L_2$, subcontracting operates at rate $d \cdot \tau$, and so a contract must be needed to regulate the appropriate amount of demand (d·τ) that subcontracting will supply during such interval. In our model, this amount (d·τ), is properly defined through optimization techniques to guarantee that the given context remains profitable for the decision maker. Such production scenarios are encountered in real cases as the one studied by Assid et al. (2015) and Dror et al. (2009), who addressed subcontracting policies under uncertainties and lack of production capacity. Nevertheless, they completely disregarded the influence of deterioration in their results. Table 2 also indicates that when $a > L_2$, the machine is so deteriorated that it must be completely stopped to be sent to preventive maintenance, which resets its age to zero. Meanwhile all demand (d) is assumed by the subcontractor. The subcontractor is no longer needed after the preventive maintenance, which restores the machine to its initial condition.

To sum up, our joint control policy is completely defined by the expressions of Table 2, and with the control parameters $(Z_0, L_0, \tau, L_1, L_2)$. Unfortunately, a shortcoming of the numerical methods is that their application is too-time consuming at the operational level, since their accuracy depends on the discrete grid steps (h_x and h_a) used in the numerical methods, as reported in Berthaut et al. (2010). Additionally, a serious drawback of the stochastic optimal control model is that it is not possible to optimize the fraction of demand satisfied by subcontracting τ. Because of the *maximum principle,* the solution for the decision variables takes only an extreme value in the set of admissible decisions $\Gamma(\alpha)$, and so fractions are disregarded. Thus, τ is considered as a given value in the stochastic optimal control model presented in Section 3.

5. Simulation-optimization approach

In order to find the optimal values of the control policy parameters, given the limitations of the traditional optimal control theory, we use a simulation-optimization approach that combines discrete-continuous simulation modelling with statistical analysis and optimization techniques. The technical advantage of our approach is that it allows us to optimize the contribution of subcontracting τ required to satisfy product demand. Another advantage of our approach is that it does not require an assumption of the continuity of the *IF* for the rate of defective and failures, as presented in Figure 2. Additionally, with the simulation approach, we are not limited to assuming Markovian dynamics, since we can consider any probability distribution for the transition rates.

5.1 Description of thfiguiree approach

The proposed simulation-optimization approach has been successfully applied to solve problems that are analytically intractable, (see Gharbi et al. (2011)), as it is the case of our model. The solving steps of our approach, presented in Figure 5 consists of the following:

- *Step 1. Mathematical formulation:* implies the representation of the production planning, subcontracting and preventive maintenance scheduling problem through a stochastic dynamic programing model based on control theory, as presented in Section 3.4. This provides a rigorous statement for the dynamics, state, decision variables, the HJB equations, the expected total cost and problem constraints.
- *Step 2. Numerical methods:* it consists in the numerical solution of the HJB Equations from the problem statement of the previous step, as detailed in Section 4.

The procedure to obtain the HJB Equations is presented in Section 3.4. The solution of the HJB Equations is fundamental to determine the structure of the optimal control policy.

- *Step 3. Control factors:* in this step we determine the control factors for production, subcontracting and preventive maintenance planning defining the obtained joint control policy $(Z_0, L_0, \tau, L_1, L_2)$.
- *Step 4. Simulation modeling:* a simulation model is developed to accurately reproduce the stochastic and dynamic behavior of the manufacturing system under analysis. The simulation model presented in the next section (5.2) uses the control policy defined in the previous step as input for conducting a number of experiments to evaluate the performance of the production system. Thus, for given values of the control parameters, the total incurred cost and a set of quality performance indices are obtained from the simulation model.
- *Step 5. Optimization:* a sequential procedure comprising design of experiments (DOE) and response surface methodology (RSM), is employed to exhaustively explore the admissible experimental domain, determine the significative control factors, fit the total incurred costs with a regression model, and determine the optimal values of the control parameters.
- *Step 6. The near-optimal control policy:* the application of the proposed simulation-optimization approach determines the production, subcontracting and preventive maintenance rates described in Table 2 for the best values of the control factors $(Z_0^*, L_0{}^*, \tau^*, L_1{}^*, L_2{}^*)$.

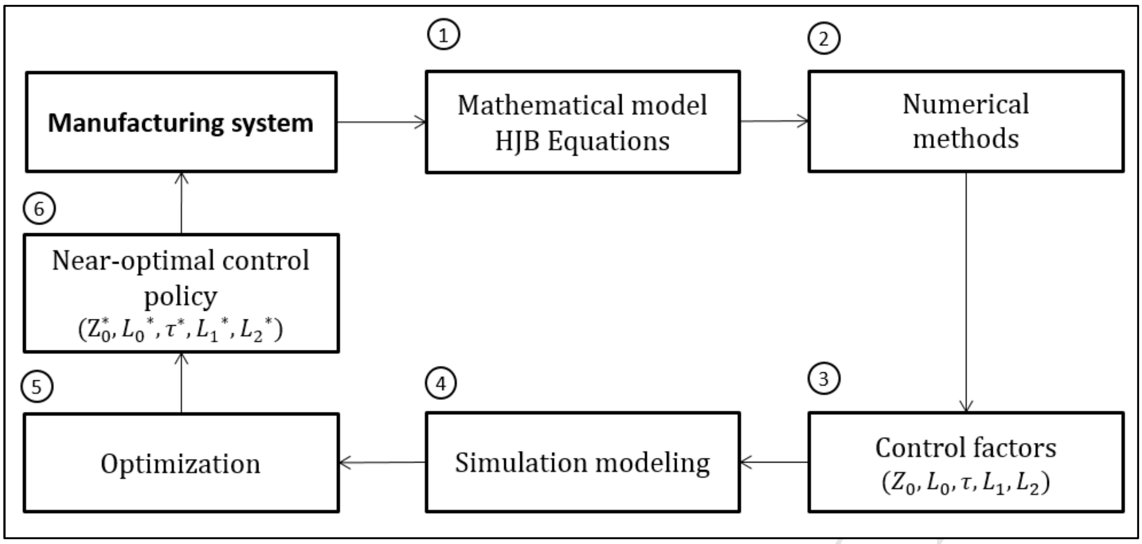


Figure 5: Solving steps of the simulation-optimization approach

5.2 Simulation model

In dealing with this kind of problems, it is usually convenient to develop a combined discrete-continuous simulation model, since it is a highly flexible option, and it permit to accurately reproduce the complex and stochastic behavior of the production system under consideration. We use the simulation software Arena complemented with C++ subroutines, comprising a number of modules which describe a specific activity or event in the system. The simulation model is represented by the block diagram of Figure 6, where we can clearly see the strong relation between the different modules of the simulation model, illustrating the amount of information that is updated in the model at each time instant.

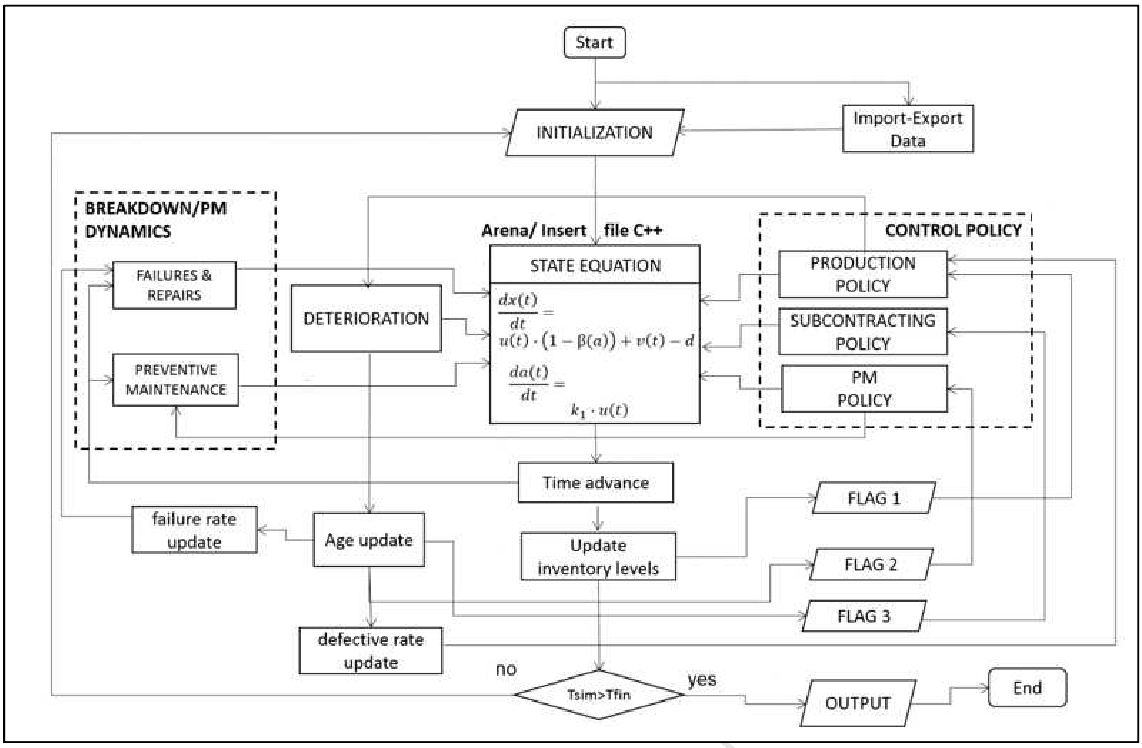


Figure 6: Simulation block diagram

5.3 Validation of the simulation model

We applied a set of verification and validation techniques, based on dynamic testing, event and operational validity to ensure that our simulation model represents accurate system behavior, as suggested by Banks et al. (2009). The values of various performance measures are shown graphically to determine whether they behave correctly, as described in the equations of Table 2. In that regard, Figure 7 represents a sample of the dynamics of the simulation model for a working year, where the time unit is defined as days, the control parameters are set to $Z_0 = 20, L_0$=19.5, $\tau = 0.5$, $L_1 = 20$ and $L_2 = 25$. At time $t = 0$, $(a = 0)$, arrow ① shows the machine in AGAN conditions, meaning that it works at the demand rate $u(t) = d,$ to maintain the stock level in its optimal value $Z_0 = 20$. After that, it experiences several random failures, and at time $t = 100$, $(a = 12)$, arrow ② shows the machine operating at rate $u(t) = d/[1 - \beta(a)]$, to compensate for the increase in the rate of defectives caused by the deterioration process. At this point, subcontracting is not required. Then, at time

$t = 160$ $(a = 20)$, arrow ③ shows that the age of the machine $a(t)$ has reached the value of L_1, hence triggering subcontracting activities with a rate of $v(t) = d \cdot \tau$. We note that while subcontracting is required the stock level decreases to $x(t) = 0$, as indicated by arrow ④, since it is assumed that subcontracting is always reliable, and so there is no need to maintain stock and there is no production, arrow ⑤. With $\tau = 0.5$, only 50% of the product demand is satisfied by subcontracting, while the remaining 50% comprises flawless and non-conforming products. Then, at time $t = 195$, $(a = 22)$ the level of deterioration of the machine is considerably high, as observed in the failure and defectives rates, arrow ⑥. Thus, subcontracting satisfies the whole product demand, as highlighted by arrow ⑦. At the same time, preventive maintenance is conducted, as shown by arrow ⑧, hence, $\theta(t) = \bar{\theta}$. Then at time $t = 200$, preventive maintenance is completed, and we have $(a = 0)$ once again, and so the failure rate and the rate of defectives are restored to AGAN conditions, as shown by arrow ⑨. Notice that the system is restored after 200 working days. With the performance of preventive maintenance, the deterioration cycle reinitiates. At time $t = 215$, $(a = 2)$ the stock level is under the threshold $Z_0 = 20$, and so, the production unit works at its maximum rate $u(t) = \bar{u}$, as shown by arrow ⑩. Then at time $t = 225$, $(a = 4)$ the system has reached the threshold Z_0, thus it operates at the demand rate $u(t) = d$, as noted by arrow ⑪. From this point, the production system follows its habitual deteriorating process.

Based on the assessment of the operational graphics presented in Figure 7, we verify that our simulation model is an accurate representation of the production system under study, and that it properly reproduces its dynamics and the twofold effect of deterioration. In the next sections, we use this simulation model to conduct a statistical analysis and parameter optimization. We will also conduct sensitivity analyses through numerical examples.

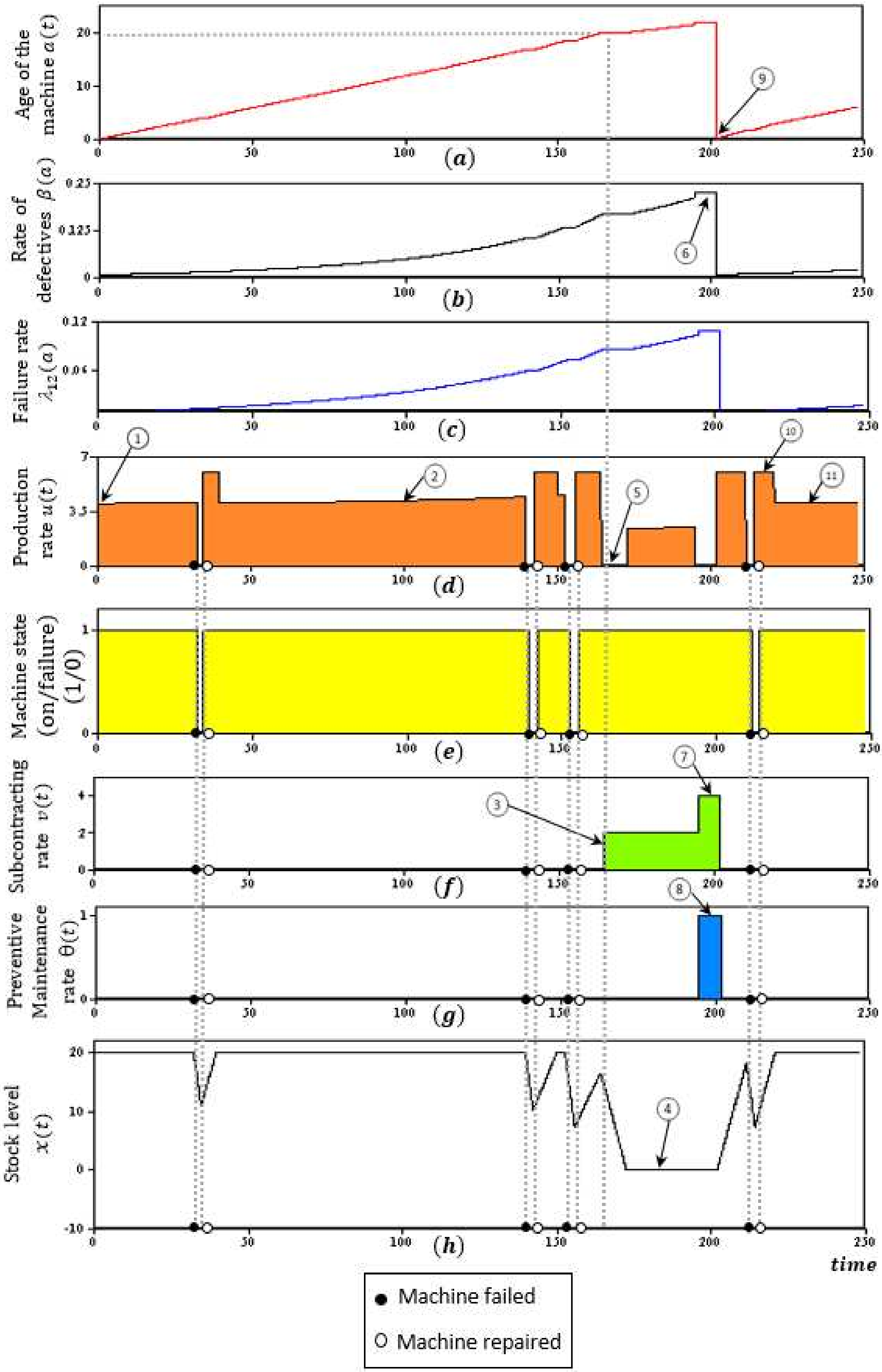


Figure 7: Dynamics of the simulation model, for $r_d = 1.16$ and $r_q = 1.097$

6. Numerical example

6.1 Integration of the subcontracting supply as a decision variable

As stated earlier, one of the objectives of the model is to optimize the fraction of demand satisfied by subcontracting τ. In Table 2, we observe that subcontracting fulfills a fraction τ of the product demand ($0 \le \tau \le 1$) during the period $L_1 < a \le L_2$. Hence, a contract is needed to establish the amount of product $d \cdot \tau$ satisfied by subcontracting during such periods. Unfortunately, in the numerical method, τ is a given constant. We should recall that a design parameter is directly dependent on whether or not it is penalized in the objective function. In our case, the subcontracting cost is considered in function (9), and this cost directly penalizes the subcontracting rate. Hence, we conjecture that the optimization of the fraction of demand satisfied by subcontracting τ, through our resolution approach could influence the performance of the manufacturing system. The control parameters of the joint policy are illustrated in Figure 4b. Additionally, in the next sections, we will analyze the case in which the control parameter τ is considered as a decision variable.

6.2 Reduction of a control parameter

The true usefulness of our simulation-optimization endeavor is illustrated with a numerical example. We note that we can simplify the procedure to determine the optimal value of the control parameters $(Z_0, L_0, \tau, L_1, L_2)$. In particular, we can reduce the number of parameters by considering condition (7). For convenience, regarding the subcontracting policy, it is possible to directly determine the control parameter L_1, given that we can know the trend of deterioration at using Equation (3) and Equation (4). Formally, this means that from condition (7), it is possible to know the exact age after which the production system is no longer capable of satisfying the product demand (age L_1). In other words, we can define the age *a(t)* when: $\bar{u} \cdot [1 - \beta(a)] \cdot \boldsymbol{\pi}_1(a) = d$. Based on this consideration, the number of control parameters is reduced to (Z_0, L_0, τ, L_2).

6.3 Numerical instance

Based on the previous discussion, the original problem reduces to define only four control factors (Z_0, L_0, τ, L_2), leading to the application of a complete factorial design 3^4. If we replicate this design three times, we will need $(3^4 x 3) = 243$ simulation runs to fully characterize our joint control policy. Simulation runs are therefore conducted according to a complete factorial design 3^4 to fit a cost function, where the simulation horizon for each replication is set to 200,000 time units to ensure steady state conditions. Table 3 presents the cost parameters used in the numerical instance, with the remaining of parameters defined as previously shown in Table 1.

Parameter:	c^+ ($/products/ day)	c^- ($/products/ day)	c_{pro} ($/product)	c_{sub} ($/product)	c_{pm} ($/pm)	c_d ($/product)	c_r ($/repair)
Value:	2.6	28	10	45	1000	20	400

Table 3. Cost parameters for the statistical analysis

The statistical analysis is conducted through an analysis of variance (ANOVA), where we define four independent variables (Z_0, L_0, τ, L_2) and one dependent variable (*total cost*). To facilitate the procedure for determining the effects of the independent variables, we define $L_0 = k \cdot L_2$, where $k \epsilon [0,1]$, thereby ensuring that $L_0 < L_2$, as observed in Figure 4b. From off-line simulations, we define the values of the independent variables as presented in Table 4. The ANOVA is conducted through the statistical software STATGRAPHICS.

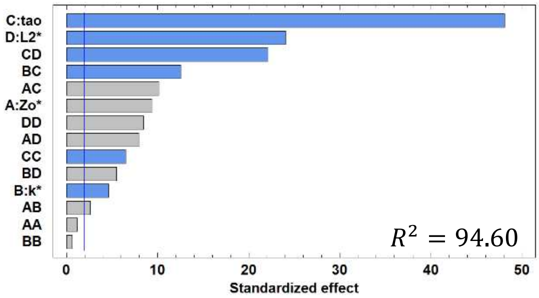
Factor	Low level	High level	Description
Z_0	5	30	Production threshold of the machine
k	0.84	0.99	Preventive maintenance variable
τ	0.30	0.95	Fraction of demand satisfied by subcontracting
L_2	21	30	Stoppage age of the machine

Table 4. Cost parameters for the statistical analysis

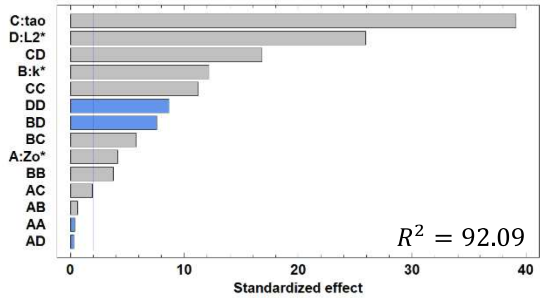
Three responses are analyzed from the 3^4 design, with the aim of obtaining uniformity of the variance and increasing the coefficient of correlation R^2, as in Lavoie et al. (2010). In our case, one surface estimates the average inventory and backlog, the second surface is for the average in-house production and subcontracting, and the third one refers to the average maintenance and defectives. Such responses have the following form:

$$Y \approx \beta_0 + \sum_i \beta_i X_i + \sum_i \sum_j \beta_{ij} X_i X_j \tag{17}$$

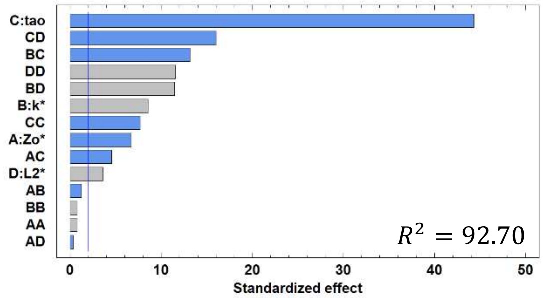
Where Y denotes the estimated responce, and X_i indicates the control factors multiplied by their estimated β_i coefficients. The responses are then multiplied by the unit costs, $(c^+,\ c^-, c_r, c_{pm}, c_d, c_{pro}, c_{sub})$ respectively, and the functions are added together. The optimal configuration cost estimate is obtained by minimizing the resulting function with non-linear programming. In Figure 8, we present the standardized Pareto chart and the R^2 coefficients for the responses considered.



a) Y_1 for inventory and backlog with ln transformation



b) Y_2 for production and subcontracting



c) Y_3 for maintenance and quality

Figure 8: Standardized Pareto charts

A logarithm transformation was used in the first response to increase the R^2 coefficient, obtaining a value of $R^2 = 94.60$. Meanwhile, no transformation was needed in the second and third responses where the values of R^2 were 92.09 and 92.70, respectively. These results indicate that a high percentage of the variability of each response is explained by the models. The obtained response functions are:

$$Y_1(Z_0, k, \tau, L_2) = 19.1405 - 0.167 \cdot Z_0 - 10.8554 \cdot k + 17.8182 \cdot \tau - 0.945252 \cdot L_2 + 0.000240198 \cdot Z_o^2 + 0.0626173 \cdot Z_0 k + 0.0556125 \cdot Z_0 \cdot \tau + 0.00314239 \cdot Z_0 L_2 + 3.5775 \cdot k^2 - 11.4568 \cdot k \cdot \tau + 0.362963 \cdot k \cdot L_2 - 1.93265 \cdot \tau^2 - 0.336562 \cdot \tau \cdot {L_2}^2 + 0.0131657 \cdot {L_2}^2 \quad (18)$$

$$Y_2(Z_0, k, \tau, L_2) = 31.0798 - 0.449675 \cdot Z_0 - 1368.98 \cdot k - 700.455 \cdot \tau + 56.4258 \cdot L_2 - 0.00404188 \cdot Z_o^2 + 0.775012 \cdot Z_0 k + 0.541949 \cdot Z_0 \cdot \tau - 0.00566091 \cdot Z_0 L_2 + 1092.07 \cdot k^2 + 272.638 \cdot k \cdot \tau - 25.9111 \cdot k \cdot L_2 + 173.168 \cdot \tau^2 + 13.2738 \cdot \tau \cdot {L_2}^2 - 0.695961 \cdot {L_2}^2 \quad (19)$$

$$Y_3(Z_0, k, \tau, L_2) = 300.294 + 0.388253 \cdot Z_0 - 217.495 \cdot k + 227.385 \cdot \tau - 18.1678 \cdot L_2 + 0.00191052 \cdot Z_o^2 - 0.366914 \cdot Z_0 k - 0.313983 \cdot Z_0 \cdot \tau - 0.00181399 \cdot Z_0 L_2 + 55.6049 \cdot k^2 - 151.521 \cdot k \cdot \tau + 9.53086 \cdot k \cdot L_2 - 28.9091 \cdot \tau^2 - 3.0716 \cdot \tau \cdot {L_2}^2 + 0.227078 \cdot {L_2}^2 \quad (20)$$

Equations (18)-(20) are then added together, and the resulting total cost function is minimized. The optimal values of the control parameters, the total cost and the cross-check validation from 50 extra-replications are presented in Table 5.

	Z_0^*	k^*	τ^*	$L_0{}^*$	$L_1{}^*$	$L_2{}^*$	Total cost estimate (C^*)	Cross-check CI (95%)
Factor	22.74	0.871	0.635	19.25	20	22.11	158.50	$[154.97, 160.48]$

Table 5. Optimal control parameters and cross-check validation

The value of $k^* = 0.871$ yields to define $L_0{}^* = 19.25$, and based on the data of Table 1 and condition (7) the age required for triggering subcontracting is $L_1{}^*$ =20. These values $(Z_0^*, L_0{}^*, \tau^*, L_1{}^*, L_2{}^*)$ are the best parameters for our joint control policy, which controls the production, subcontracting, and preventive maintenance rate simultaneously, at a minimum cost. Based on this statistical analysis, we can state that our simulation-optimization approach determines the values of the control parameters adequately, and that the total cost given by the second-order model is appropriate for application in this case.

7. Policy implementation

The technical advantage of the proposed joint control policy is based on the fact that it serves to operate the manufacturing system more smoothly and predictably. Moreover, its implementation is facilitated at considering Equation (2), since at combining with the data of Table 1, the obtained control parameter $L_0{}^* = 19.25$ implies that the production system must produce at least $N_0{}^* = 664$ parts to justify the conduction of preventive maintenance. Furthermore, at age $L_1{}^*$ =20, the system has to produce $N_1{}^* = 690$ parts to trigger subcontracting, and if preventive maintenance has not been conducted at age $L_2^* = 22.11$, then $N_2^* = 762$ parts are needed to stop production. In this sense, the quantities(N_0^*, N_1^*, N_2^*), provides a more physical definition of the control parameters in terms of the number of parts produced (denoted by "n"). Additionally, to properly implement the control policy, we must continuously monitor the stock level and the number of parts produced. For instance, at considering the optimal parameters of Table 5, Figure 9 illustrates a sample of the decision-making process for the case when $(x, n) = (-5{,}700)$, where the blond boxes indicate the appropriate control rates for the illustration.

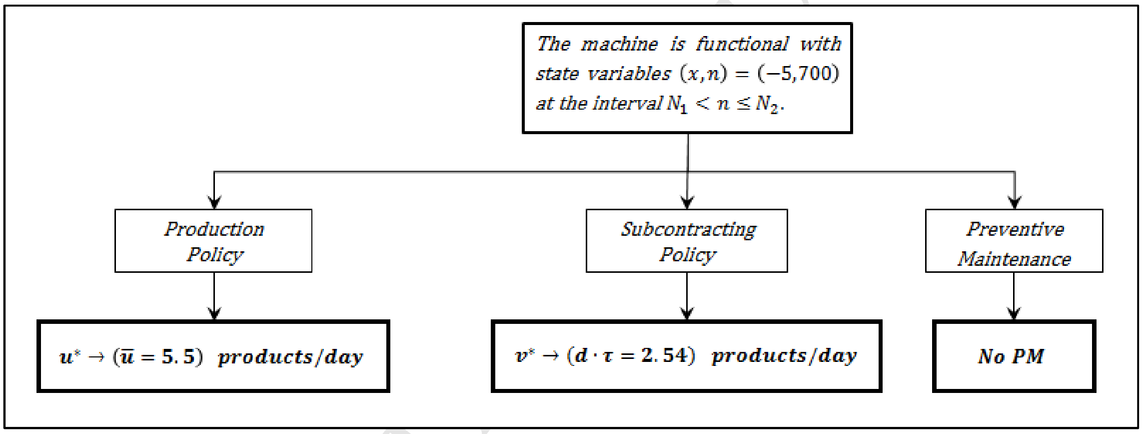


Figure 9: Sample of the policy implementation

8. Sensitivity analysis

A sensitivity analysis based on a series of numerical examples was conducted with the aim of increasing the operational validity of our model, and confirm the effectiveness of the obtained joint control policy. In particular, we studied the sensitivity of the joint control policy according to variations of different costs scenarios and changes in increment of the rate of defectives, as well as the failure intensity.

8.1 Effect of cost variations

Several cost categories were considered in the sensitivity analysis to gain insight into the behavior of the production system and assess our simulation control approach. The numerical example discussed previously was used to perform a set of cost scenarios presented in Table 6, where we highlight the strong relationship between cost variations and control parameters $\left(Z_0^*, {N_0}^*, \tau^*, {N_1}^*, {N_2}^*\right)$ and their respective incurred costs. The basic case of Table 6 was obtained with $r_d = 1.109$ and $r_q = 1.106$, leading us to define ${L_1}^* = 20$ from condition (7), implying that ${N_1}^* = 690$ parts. Moreover, in Table 6 we include complementary performance indices such as the long-run average quantity per unit of time of items produced by the machine, Q_{pro}^*; the long-run average quantity per unit of time of items supplied by subcontracting, Q_{sub}^*; and the long-run average quantity per unit of time of defectives, Q_{def}^*. These indices were obtained from the simulation model when the optimal solutions were applied. The following variations are analyzed and compared to the basic case:

Cases	Par.	Cost variations							Optimal parameters variations								Total Cost	Remark
		c^+	c^-	c_d	C_{pro}	C_{sub}	c_{pm}	c_r	Z_0^*	k^*	τ^*	${N_0}^*$	${N_2}^*$	Q_{pro}^*	Q_{sub}^*	Q_{def}^*	C^*	
Basic case	-	2.6	28	20	10	45	1000	400	22.74	0.871	0.635	664	762	4.027	0.096	0.123	158.50	Base for the comparison
Case I	c^+	***2***	28	20	10	45	1000	400	25.65	0.868	0.627	635	732	4.095	0.034	0.129	156.73	$Z_0^* \uparrow, {N_0}^* \downarrow, \tau^* \downarrow, {N_2}^* \downarrow$
Case II		***3.6***	28	20	10	45	1000	400	7.89	0.884	0.676	779	881	3.128	0.993	0.121	166.27	$Z_0^* \downarrow, {N_0}^* \uparrow, \tau^* \uparrow, {N_2}^* \uparrow$
Case III	c^-	2.6	***20***	20	10	45	1000	400	19.38	0.872	0.627	640	734	4.091	0.037	0.128	154.54	$Z_0^* \downarrow, {N_0}^* \downarrow, \tau^* \downarrow, {N_2}^* \downarrow$
Case IV		2.6	***38***	20	10	45	1000	400	27.90	0.870	0.659	682	784	3.885	0.235	0.120	164.11	$Z_0^* \uparrow, {N_0}^* \uparrow, \tau^* \uparrow, {N_2}^* \uparrow$
Case V	c_d	2.6	28	***10***	10	45	1000	400	24.28	0.870	0.624	634	730	4.092	0.038	0.130	146.74	$Z_0^* \uparrow, {N_0}^* \downarrow, \tau^* \downarrow, {N_2}^* \downarrow$
Case VI		2.6	28	***50***	10	45	1000	400	21.72	0.873	0.688	702	804	3.316	0.791	0.107	192.43	$Z_0^* \downarrow, {N_0}^* \uparrow, \tau^* \uparrow, {N_2}^* \uparrow$
Case VII	C_{pro}	2.6	28	20	***5***	40	1000	400	24.70	0.869	0.625	632	728	4.094	0.034	0.128	153.38	$Z_0^* \uparrow, {N_0}^* \downarrow, \tau^* \downarrow, {N_2}^* \downarrow$
Case VIII		2.6	28	20	***20***	40	1000	400	15.43	0.893	0.745	798	894	2.664	1.444	0.108	190.09	$Z_0^* \downarrow, {N_0}^* \uparrow, \tau^* \uparrow, {N_2}^* \uparrow$
Case IX	C_{sub}	2.6	28	20	10	***35***	1000	400	13.35	0.900	0.775	820	912	2.471	1.635	0.106	148.41	$Z_0^* \downarrow, {N_0}^* \uparrow, \tau^* \uparrow, {N_2}^* \uparrow$
Case X		2.6	28	20	10	***50***	1000	400	24.94	0.868	0.625	630	726	4.098	0.031	0.129	159.49	$Z_0^* \uparrow, {N_0}^* \downarrow, \tau^* \downarrow, {N_2}^* \downarrow$
Case XI	c_{pm}	2.6	28	20	10	45	***700***	400	24.69	0.869	0.626	633	729	4.092	0.038	0.130	155.48	$Z_0^* \uparrow, {N_0}^* \downarrow, \tau^* \downarrow, {N_2}^* \downarrow$
Case XII		2.6	28	20	10	45	***1700***	400	17.09	0.881	0.676	755	857	3.078	1.035	0.112	169.54	$Z_0^* \downarrow, {N_0}^* \uparrow, \tau^* \uparrow, {N_2}^* \uparrow$
Case XIII	c_r	2.6	28	20	10	45	1000	***50***	18.49	0.878	0.661	732	834	3.193	0.917	0.111	151.88	$Z_0^* \downarrow, {N_0}^* \uparrow, \tau^* \uparrow, {N_2}^* \uparrow$
Case XIV		2.6	28	20	10	45	1000	***600***	24.44	0.869	0.628	637	733	4.090	0.038	0.128	160.26	$Z_0^* \uparrow, {N_0}^* \downarrow, \tau^* \downarrow, {N_2}^* \downarrow$

Table 6. Sensitivity analysis for different cost variations

- *Variation of the Inventory cost, c^+ (case I and II):* When the inventory cost c^+ increases (case II), the production threshold Z_0^* decreases, because the inventory is more penalized. Further, this threshold reduction also decreases the preventive maintenance zone, consequently increasing ${N_0}^*$, because the machine spends less time operating at its maximum rate; this leads to less deterioration, meaning that less preventive maintenance is needed. At increasing ${N_0}^*$, the machine operates for a longer time period before preventive maintenance is conducted, and then the subcontracting supply τ^* increases to compensate for the amount of defectives generated; as well subcontracting, contributes more, increasing Q_{sub}^*. At increasing c^+, the machine is less utilized at its maximum rate, which allows it to operate for longer periods (increasing ${N_2}^*$) before it stops. Additionally, in the long-run the machine satisfies less product demand, decreasing Q_{pro}^*, and so the average of defectives, Q_{def}^* decreases. It should be noted that a lower inventory cost produces the opposite effects (case I).
- *Variation of the Backlog cost, c^- (case III and IV):* When we increase c^- (case IV), the model reacts by increasing the production threshold Z_0^* because the product backlog is more penalized, and so we need more stock to palliate shortages. Nevertheless, with a more penalized backlog, more subcontracting is needed, thus increasing τ^* since it is assumed that subcontacting is always reliable and does not fail. Also, at increasing c^- the machine remains operational for longer before preventive maintenance is conducted, with more subcontracting used. Therefore, ${N_0}^*$ and ${N_2}^*$, increases. With preventive maintenance delay, subcontracting contributes more to the total demand in the long-run, thereby increasing Q_{sub}^*. Thus, the contribution of the machine Q_{pro}^* decreases, leading to a lower production of defectives Q_{def}^*. A decrease in the backlog cost has the opposite effects (case III).

- *Variation of the Defectives cost, c_d (case V and VI):* At increasing c_d (case VI), the need for a more reliable systems rises, and so this increments τ^* and Q^*_{sub}, because subcontracting is assumed to supply only flawless products. Meanwhile, the production threshold Z^*_0 decreases mainly because with more subcontracting the system is more reliable. Also with more subcontracting participation, the long-run average of defectives, Q^*_{def}, decreases. Moreover, an increment of the defectives cost c_d leads to decrease the contribution of the machine Q^*_{pro}, because the machine is less used, and so its deterioration decreases and this delays the performance of preventive maintenance (thus increasing ${N_0}^*$) to allow the machine to be operated for longer, and this also increases ${N_2}^*$. The opposite occurs when $\mathrm{c_d}$ decreases (case V).
- *Variation of the Production cost, c_{pro} (case VII and VIII):* With an increase of c_{pro} (case VIII), it is normal to see an increase in the use of subcontracting τ^*, and this implies a more reliable system because subcontracting is assumed to be free of defectives and failures, thus the production threshold Z^*_0 decreases. Consequently, with an increasing c_{pro}, it is logical to expect that the machine spends less time operating, deteriorating less, and so ${N_0}^*$ and ${N_2}^*$ increase, and less preventive maintenance is conducted. With a higher production cost, it is normal for the contribution of the machine Q^*_{pro} to drop, and so the long-run average of defectives, Q^*_{def}, decreases, while the contribution of subcontracting to the total demand, Q^*_{sub}, increases considerably to compensate. The inverse occurs when the production cost decreases (case VII).
- *Variation of the Subcontracting cost, c_{sub} (case IX and X):* We notice that when we increase C_{sub} (case X), it reduces the use of subcontracting, thus decreasing τ^* and Q^*_{sub}, and leaving us with a less reliable system. In this context, the system protects

itself against backlogs by increasing Z_0^*, and as the machine is more utilized, its deterioration pace accelerates, and more preventive maintenance is needed, then decreasing ${N_0}^*$. Also, ${N_2}^*$ decreases, since the machine deteriorates faster when it operates with a higher threshold Z_0^*, and it must be stopped earlier because its operation is not profitable. Furthermore, with a higher subcontracting cost, it is normal for its contribution to the total demand Q_{sub}^* to decrease, and hence, the long-run average of the units produced by the machine Q_{pro}^*, increases instead, and with this increment, the quantity of defectives Q_{def}^*, also increases. A decrease in the subcontracting cost has the contrary effects (case IX).

- *Variation of the Preventive maintenance cost, c_{pm} (case XI and XII):* At increasing c_{pm} (case XII), it is normal to delay the performance of preventive maintenance, and thus ${N_0}^*$ increases. Consequently with less preventive maintenance, subcontracting fulfills a higher fraction of product demand, hence increasing τ^*. Since subcontracting is free of failures and defectives, we have a more reliable system that decreases the threshold Z_0^*. Additionally, because of the reduction of Z_0^*, the machine spends less time operating at its maximum rate, deteriorates less, and thus increases its stoppage age ${N_2}^*$, in addition to reducing its contribution to the total demand Q_{pro}^*. With this reduction, subcontracting contributes more, hence increasing Q_{sub}^*, which this leads to a reduction in the long-run average of defectives Q_{def}^*. A reduction of the preventive maintenance cost has the inverse effects (case XI).
- *Variation of the Repair cost, c_r (case XIII and XIV):* An increase in the repair cost $\mathrm{c_r}$, (case XIV) promotes the conduct of more preventive maintenance, hence ${N_0}^*$ decreases; we have less need for subcontracting, and thus, Q_{sub}^* and τ^*decrease. With a reduction in the subcontracting supply, we have a less reliable system that

protects itself against backlogs and defectives, thereby increasing the production threshold Z_0^*. With this increment, the machine spends more time operating at its maximum rate, deteriorates faster, and the stoppage age ${N_2}^*$decreases. With increased repair costs, the contribution of the machine Q_{pro}^* increases, and consequently, the long-run average of defectives Q_{def}^* increases as well. The opposite effects occur when we decrease the repair cost (case XIII).

This sensitivity analysis assesses our resolution approach, and corroborates the structure of the joint control policy and parameters obtained. In the next section, we will complement the sensitivity analysis with an examination of the effect of two system parameters, namely, the variation of the rate of defectives and the failure frequency.

8.2 Effect of the rate of defectives and failure rate parameters

Two more issues remain to be addressed in the sensitivity analysis to provide a better understanding of the control factors $(Z_0^*, {N_0}^*, \tau^*, {N_1}^*, {N_2}^*)$ when varying two system adjustment parameters. Therefore, in this section, an extra set of simulation runs are conducted to study the sensitivity of the control parameters with respect to variations of r_d and r_q. Table 7 presents the results of four configurations tested, where the value of the remaining parameters were defined, as in Table 1. The influence of the variation of r_d and r_q is discussed as follows:

- *Variation of the adjustment parameter r_d (case i and ii):* before examining this parameter, we must recall that the role of r_d is to modify the pace of deterioration of the machine (as illustrated in Figure 2). Particularly, when we increase r_d (*case ii*), we accelerate the trend of deterioration of the machine, then generating more defectives and this decreases ${N_1}^*$, also, this promotes the use of more subcontracting, hence increasing Q_{sub}^* and τ^*. With more subcontracting participation we have a more reliable system, and thus the production threshold Z_0^* decreases. Furthermore, with the increment of r_d, the machine deteriorates faster, which decreases ${N_2}^*$, and this reduces ${N_0}^*$. At increasing r_d, the contribution of the machine Q_{prod}^* decreases while the long-run average of defectives, Q_{def}^*, increases, because the system produces defectives at a higher rate. We observe the inverse effects when r_d decreases (*case i*).

- *Variation of the adjustment parameter* r_q *(case iii and iv)*: the effect of this parameter is explained mainly with the concept of failure intensity. For the case where we increase r_q (case iv), the machine increases its failure rate, and so it breaks down more often. With more frequent failures, the system reacts by protecting against backlogs, increasing the production threshold Z_0^* and mainly requiring more subcontracting supply, this reduces ${N_1}^*$ and increases τ^* and Q_{sub}^* in order to ensure that product demand is satisfied. Further, with the increment of r_q, the stoppage age ${N_2}^*$ decreases, since the machine deteriorates faster, and this also reduces age ${N_0}^*$, favoring a sooner conduction of preventive maintenance. Other parameters move as predicted from a practical point of view, in order to avoid further shortages and defectives. At increasing r_q the contribution of the machine Q_{prod}^* decreases, because it is less reliable, and has more frequent failures, and so the average of defectives Q_{def}^* also decreases. The contrary occurs when r_q decreases (case iii).

We end the sensitivity analysis by observing that our approach represents an effective solution alternative, and that its operational validity has been assessed through different cost and parameter configurations.

Cases	Parameter's variations r_d	r_q	Z_0^*	k^*	${N_0}^*$	τ^*	${N_1}^*$	${N_2}^*$	Q_{prod}^*	Q_{sub}^*	Q_{def}^*	Total Cost C^*	Remark
			Optimal control parameters variations										
Basic case	1.109	1.106	22.74	0.871	664	0.635	690	762	4.027	0.096	0.123	158.50	Base for the comparison
Sensitivity of the common ratio r_d													
Case i	***1.05***	1.106	23.56	0.840	684	0.594	759	814	4.041	0.024	0.065	141.79	$Z_0^* \uparrow, {N_0}^* \uparrow, \tau^* \downarrow, {N_1}^* \uparrow, {N_2}^* \uparrow$
Case ii	***1.13***	1.106	9.72	0.899	641	0.675	655	713	3.952	0.191	0.143	156.44	$Z_0^* \downarrow, {N_0}^* \downarrow, \tau^* \uparrow, {N_1}^* \downarrow, {N_2}^* \downarrow$
Sensitivity of the common ratio r_q													
Case iii	1.109	***1.10***	18.06	0.847	727	0.628	724	859	4.083	0.057	0.140	159.25	$Z_0^* \downarrow, {N_0}^* \uparrow, \tau^* \downarrow, {N_1}^* \uparrow, {N_2}^* \uparrow$
Case iv	1.109	***1.12***	24.35	0.871	648	0.652	655	732	3.862	0.250	0.112	151.16	$Z_0^* \uparrow, {N_0}^* \downarrow, \tau^* \uparrow, {N_1}^* \downarrow, {N_2}^* \downarrow$

Table 7. Parameters for the statistical analysis

9. Comparative study

The joint production, subcontracting and maintenance policies proposed in this paper has not been addressed under the same assumptions in the literature yet. The closest works to our model only consider production and preventive maintenances strategies, where some of them integrates system deterioration as in Bouslah et al. (2016), while others do not consider such deteriorations like in Berthaut et al. (2010). Given the absence of an analytic solution, a comparative study is performed to show that our proposed policy where subtracting is an option (*sub*) outperforms that in the literature where subcontracting is not an option (*no-sub*). In particular, we will compare the optimal total incurred cost, obtained from our joint control policy (C^*_{sub}) with the optimal total cost (C^*_{no-sub}) derived from a policy based on Berthaut et al. (2010) Bouslah et al. (2016) and considering the effects of deterioration, but with the main difference that subcontracting parameters are completely disregarded. In such policy the optimization of C^*_{no-sub} is limited to the parameters of the production threshold, the age required to conduct preventive maintenance and the age to stop the machine (Z_o, L_o, L_2).

We present in Table 8 the optimal total incurred cost C^*_{sub} and C^*_{no-sub} for all the sensitivity analysis cases of Tables 6 and 7. The results presented in Table 8, were obtained under the same conditions (simulation length, experimental domain, etc.) following the same simulation optimization approach used in previous sections and with the data parameters shown in Table 3. The results presented in Table 8 clearly show that for all the cases, the optimal total incurred cost (C^*_{sub}) considering the joint production, subcontracting and preventive maintenance policies is always inferior in the range of [10.94% - 20.21%] than the optimal total cost C^*_{no-sub} where subcontracting is not considered.

Cases	Optimal total cost		Optimal total cost differences
	C^*_{sub}	(C^*_{no-sub})	$\Delta\, C^*_{sub}/(C^*_{no-sub})$
Basic case	158.20	180.22	12.21%
Sensitivity for the cases of Table 6			
Case I	156.73	175.99	10.94%
Case II	166.27	192.13	13.45%
Case III	154.54	177.00	12.68%
Case IV	164.11	194.89	15.79%
Case V	146.74	166.87	12.06%
Case VI	192.43	219.97	12.51%
Case VII	153.38	172.68	11.17%
Case VIII	190.09	217.90	12.76%
Case IX	148.41	177.73	16.49%
Case X	159.49	181.46	12.10%
Case XI	155.48	174.94	11.12%
Case XII	169.54	192.50	11.92%
Case XIII	151.88	173.61	12.51%
Case XIV	160.26	184.00	12.90%
Sensitivity for the cases of Table 7			
Case i	141.79	160.35	11.57%
Case ii	156.44	194.04	19.37%
Case iii	159.36	180.34	11.63%
Case iv	151.12	189.40	20.21%

Table 8. Cost differences of the comparative study

Consequently, our proposed policy leads to a lower total incurred cost compared to the case where subcontracting is disregarded.

10. Conclusions

This paper analyzed the impact of quality and reliability deterioration for an unreliable and imperfect manufacturing system, when preventive maintenance and subcontracting activities are available. We developed a stochastic optimization model taking into consideration production, subcontracting and preventive maintenance decisions and two state variables, denoted by the stock level and the age of the machine. We established optimality conditions in the form of HJB equations, and we used a finite difference scheme to approximate the continuous problem by a discrete counterpart. Additionally, a simulation-optimization approach is proposed to determine the parameters of the obtained feedback policy, which is a derivation of the HPP with preventive maintenance and subcontracting activities. We have shown that there is a strong relationship between the number of parts to hold in inventory, subcontracting, preventive maintenance parameters and deterioration. The structure of the optimal joint control policy, as well as the usefulness of the proposed approach, are illustrated and validated through a numerical example and a sensitive analysis. The obtained total cost under a joint production, subcontracting and preventive maintenance control policy was contrasted with the case where subcontracting parameters are not considered. The results show that our total cost is always inferior up to 20.21% relative to the total cost where subcontracting parameters are disregarded. A possible extension of the proposed model could involve the case where subcontracting is not always reliable, with a random proportion of defectives supply and the case of more complex manufacturing system (i.e. multiple machines).

Acknowledgements

This research has been supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) under grant number RGPIN-2015-06026.

Appendix A

The dynamics of the machine is described by a continuous time stochastic process, with transition rates from mode α to mode α' called $\lambda_{\alpha\alpha'}(a,\theta)$ with $\alpha, \alpha' \in \{1,2,3\}$. The transition diagram, describing the dynamics of the considered machine is presented in Figure A.1.

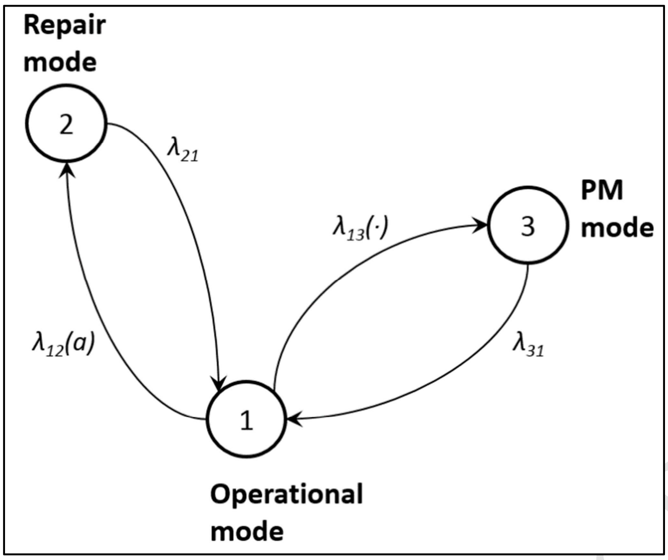


Figure A.1. State transition diagram

In order to increase the system capacity at a given deterioration level, we control the transition rate from mode 1 to 3 (i.e., $\lambda_{13} = \theta(\cdot)$) with the machine age dependent failure rate $\lambda_{12}(a)$ given by equation (4). For the considered system, the corresponding 3×3 transition matrix Q depends on $\theta(\cdot)$ and $a(t)$ and corresponds to one of an ergodic nonhomogeneous semi-Markovian process due to imperfect repairs. Hence, $\xi(t)$ is described by the following matrix:

$$Q(a,\theta) = \begin{pmatrix} \lambda_{11} & \lambda_{12}(a) & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{pmatrix} \tag{A.1}$$

where $\lambda_{13} = \theta$, $\lambda_{23} = \lambda_{32} = 0$ and $\lambda_{12}(a)$ is the increasing failure rate related to the age of the machine. The transition rates in equation (A.1) verify the following conditions:

$$\lambda_{\alpha\alpha'}(a,\theta) \geq 0, \quad (\alpha \neq \alpha') \tag{A.2}$$

$$\lambda_{\alpha\alpha'}(a,\theta) = -\sum_{\alpha \neq \alpha'} \lambda_{\alpha\alpha'}(a,\theta) \tag{A.3}$$

References

Assid, M., Gharbi, A., Dhouib. K. 2015. *Joint production and subcontracting planning of unreliable multi-facility multi-product production systems.* Omega: The International Journal of Management Science, 50, 54-69.

Banks, J., Carson, J.S., Nelson, B.L. and Nicol, D.M. 2009. *Discrete-Event System Simulation,* Prentice Hall, 5th Edition, 640p.

Berthaut, F., Gharbi, A., Kenné, J.P., Boulet, J.F. 2010. *Improved joint preventive maintenance and hedging point policy*. International Journal of Production Economics 127, 60–72.

Boukas, E.K. and Hauire, A. 1990. *Manufacturing flow control and preventive maintenance: a stochastic control approach.* IEEE Transactions on Automatic Control, 33, 1024-1031.

Bouslah, B., Gharbi, A., and Pellerin, R. 2016. *Joint economic design of production, continuous sampling inspection and preventive maintenance of a deteriorating production system*. International Journal of Production Economics, 173, 184-198.

Chelbi, A. and Ait-Kadi, D. 2004. *Analysis of a production/inventory system with randomly failing production unit submitted to regular preventive maintenance.* European Journal of Operational Research, 156:3, 712-718.

Chouiki, H., Khatab, A., rezg, N. 2014. *A condition-based maintenance policy for a production system under excessive environmental degradation.* Journal of Intelligent Manufacturing, 25:4, 727-737.

Colledani, M. and Gershwin, S.B. 2013. *A decomposition method for approximate evaluation of continuous flow multi-stage lines with general Markovian machines.* Annals of Operation Research, 209:1, 5-40.

Colledani, M., and Tolio, T. 2012. *Integrated quality, production logistics and maintenance analysis of multi-stage asynchronous manufacturing systems with degrading machines.* CIRP Annals – Manufacturing Technology, 61:1, 455-458.

Dehayem-Nodem, F.I., Kenne, J.P., and Gharbi, A. 2011a. *Production planning and repair/replacement switching policy for deteriorating manufacturing systems.* International Journal of Advanced Manufacturing Technology, 57:5-8, 827-840.

Dehayem-Nodem, F.I., Kenne, J.P. and Gharbi, A. 2011b. *Simultaneous Control of Production, Repair/Replacement and Preventive Maintenance of Deteriorating Manufacturing Systems*. International Journal of Production Economics, 134:1, 271-282.

Dror, M., Smith, K.R., and Yano, C.A. 2009. *Deux Chemicals Inc. Goes Just–in-Time*, Interfaces, 39:6, 503-515.

Gershwin, S.B. 1994. *Manufacturing Systems Engineering,* First Edition; Prentice Hall, Cambridge, Massachusetts, USA, 350 p.

Gharbi, A., Hajji, A. and Dhouib, K. 2011. *Production rate control of an unreliable manufacturing cell with adjustable capacity*. International Journal of Production Research, 49:21, 6539-6557.

Hajej, Z., Dellagi, S., Rezg, N. 2011. *Optimal integrated maintenance/production policy for randomly failing systems with variable failure rate.* International Journal of Production Research, 49:19, 5695-5712.

Hajej, Z, Rezg, N. and Gharbi, A. 2014. *Forecasting and maintenance problem under subcontracting constraint with transportation delay.* International Journal of Production Research, 52:22, 6695-6716.

Haoues, M., Dahane, M., Kenza-Mouse, N. 2016. *Outsourcing optimization in two-echelon supply chain network under integrated production-maintenance constraints.* Journal of Intelligent Manufacturing, 1-25.

Kenné, J.P., Dejax, P., and Gharbi, A. 2012. *Production planning of a hybrid manufacturing-remanufacturing system under uncertainty within a closed-loop supply chain.* International Journal of Production Economics, 135, 81-93.

Kim J. and Gershwin S. 2008. *Analysis of long flow lines with quality and operational failures.* IIE Transactions, 40, 284-296.

Khatab, A., Ait-Kadi, D., Rezg, N. 2014. *Availability optimization for stochastic degrading systems under imperfect preventive maintenance*. International Journal of Production Research, 52:14, 4132-4141.

Kouedeu, A.F., Kenne, J.P., Dejax, P., Songmene, V., Polotski, V. 2015. *Production and maintenance planning for a failure-prone deteriorating manufacturing system: a hierarchical control approach.* International Journal of Advanced Manufacturing Technology, 76:9, 1607-1619.

Kouki, M., Dellagi, S., Achour, Z., Erray, W. 2014. *Optimal integrated maintenance policy based on quality deterioration.* IEEE International conference on Industrial engineering and engineering management, (IEEM).

Kushner, H.J. and Dupuis, P.G. 1992. *Numerical Methods for Stochastic Control Problems in Continuous Time,* (Springer, New York, NY).

Lam, Y., Zhu, L.X. and Chan, J.S.K., and Liu, Q. 2004. *Analysis of data from a series of events by a geometric process model*. Acta Mathematicae Applicatae, 20, 263–282.

Lavoie, P., Gharbi, A., Kenne, J.P. 2010. *A comparative study of pull control mechanisms for unreliable homogeneous transfer lines.* International Journal of Production Economics, 124, 241-251.

Lee, H.H. 2005. *A cost/benefit model for investments in inventory and preventive maintenance in an imperfect production system*. Computers & Industrial Engineering, 48:1, 55-68.

Lee, H.H. 2008. *The investment model in preventive maintenance in multi-level production systems*. International Journal of Production Economics, 112:2, 816-828.

Love, C.E., Zhang, Z.G., Zitron, M.A., Guo, R. 2000. *A discrete semi-Markov decision model to determine the optimal repair/replacement policy under general repairs.* European Journal of Operational Research, 125, 398-409.

Lu, S., Tu, Y.C., and Lu, H. 2007. *Predictive condition-based maintenance for continuously deteriorating systems*. Quality and Reliability Engineering International, 23, 71-81.

Ouaret, S., Kenné J.P. and Gharbi A. 2018. *Production and replacement policies for a deteriorating manufacturing system under random demand and quality*. European Journal of Operational Research, 264, 623–636

Rivera-Gómez, H., Gharbi, A., Kenne, J.P., Montaño-Arango, O., and Hernandez-Gress, E.S. 2016. *Production control problem integrating overhaul and subcontracting strategies for a quality deteriorating manufacturing system.* International Journal of Production Economics, 171:1, 134-150.

Wen, D., Ershun, P., Ying, W., and Wenzhu, L. 2016. *An economic production quantity model for a deteriorating system integrated with predictive maintenance strategy*. Journal of Intelligent Manufacturing, 27:6, 1323-1333.

Highlights

- Production, maintenance and subcontracting are analyzed in an integrated model
- The deterioration process considered involves effects on quality and reliability
- The model defines control parameters though numerical techniques and simulation
- The results can be applied in industries such as pharmaceutical, automotive, etc.
- We keep a tractable state space at modeling deterioration with increasing functions