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Joint production, inspection and maintenance control policies for deteriorating system under quality constraint

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ABSTRACT

This paper studies the integration of production, sampling inspection and age-based maintenance planning for an unreliable production system subject to gradual deterioration. The deterioration process of the production unit has a twofold effect on its reliability and product quality. To mitigate the effects of such deterioration, an age-based major maintenance can be conducted, which denotes a perfect repair that restores the production unit to initial conditions. The quality control is performed through a sampling plan that inspects a fraction of the parts produced. The problem further considers that the optimal decision must be determined under a constraint on the outgoing quality required by the final customer. In this domain, standard sampling procedures are applicable only to production process that are statistically stable and under control. Nevertheless, such sampling plans disregard the interaction with production management and maintenance issues and they do not consider the effects of deterioration. In this paper a new joint control policy considering the interactions between production-quality and maintenance is proposed. A stochastic mathematical model is developed through specialized optimization techniques to solve such quality constrained problem. Numerical examples are provided to illustrate the usefulness of the proposed approach and to study the interactions between production-quality and maintenance strategies. An extensive sensitivity analysis and a comparative study are conducted to illustrate the effectiveness of the obtained joint control policy.

1. Introduction

Increasing emphasis on sustainable production requires preserving the efficiency of degrading resources over time. Additionally, maintenance planning must consider the interactions with quality control and production planning, since they are fundamental functions for economic success in the manufacturing industry. However, production systems also face the negative effects of deterioration processes. Current approaches result in sub-performing unbalance systemic solutions that tend to privilege one or two of the aspects, focusing on the production-maintenance or production-quality interactions, reducing the overall manufacturing system efficiency. Thus, integrated models are needed to determine the right balance among production-quality and maintenance functions and improve long-term system's performance. In order to provide a comprehensive context of the current literature, we analyze five research directions that have contributed to the domain of deteriorating manufacturing systems. We focus in the following: i) inspection strategies, ii) deterioration, iii) production and quality strategies, iv) production and maintenance strategies and v) integrated strategies of production-quality and maintenance.

The vast majority of practical cases of inspection strategies reported in the literature in the most diverse settings, consider that a critical issue to managers is to invest in improving process control, worker's skills in inspection, as well as appraisal activities to determine the degree of

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conformance and screen out defectives. For example, Yoo et al. [1] presented an algorithm for finding the optimal inspection policy that defines the change between entire lot screening and no inspection. Hsu and Hsu [2] addressed the problem of developing an economic production quantity model with imperfect production, inspection errors and planned backorders. They observed that defective items have a significant impact on the optimal production lot size and the backorder quantity. The model of Chen [3] assumed that preventive maintenance error results in the production of non-conforming items. They determined the optimal inspection interval, inspection frequency and production quantity that yields the maximal expected profit. Mhada et al. [4] extended inspection models by providing a fast algorithm for solving the buffer sizing and inspection positioning problem of large production lines. Ait-El-Cadi et al. [5,6] studied the joint design of production and maintenance controls under quality and reliability deterioration. They considered the case of imperfect maintenance and inspection error. As observed in the discussed papers, sampling plans can generate significant economic savings compared to 100 % inspection. However, the application of sampling plans is currently limited to stable process, additionally such plans are not conceived for deteriorating systems. Therefore, more research is needed to extend the use of sampling plans in the context of deterioration.

It is quite common in systems of production of goods and services that they experience random deterioration with respect to usage. Such deterioration process impacts not only the product quality, but also its reliability and safety. Several authors have addressed deterioration issues, as the paper developed by Kouedeu et al. [7], who introduced a joint analysis of the optimal production and maintenance planning policies. In their model, the machine's failure rate deteriorates, depending on the number of imperfect repairs. Ayed et al. [8] proposed an optimal production plan considering the degradation of the manufacturing system, which satisfies a random demand under a require service level. Oosterom et al. [9] focused on the modeling of a deteriorating system; that deteriorates over a finite set of condition states. Also, they provided a set of conditions for which they characterized the structure of the replacement policy. In the model developed by Ouaret et al. [10], an optimal production and replacement plan was investigated in terms of a manufacturing and a recovery machine. They considered that the deterioration of the manufacturing machine was caused by an aging process affecting its availability and the quality of the parts produced. Also, they assumed that the defective parts affect the failure process of the recovery machine. Ouaret et al. [11] dealt with the determination of the production rates of a manufacturing and a remanufacturing machine which deteriorates with time as a result of imperfect repairs. Also, they determined the replacement rate of the remanufacturing machine that minimize the total cost. Wang et al. [12], developed a production and maintenance model, where they assumed that when the deterioration state surpasses a predetermined failure level the machine produces defectives. In their model such items accelerate the deterioration of the downstream machines. On the basis of the discussed works, we notice that the deterioration of production systems can reduce the efficiency of its operations or affect the quality of the good or services it delivers. This evidently results in increasing operating costs. Therefore, it can be advantageous to incorporate deterioration issues on the joint production, quality and maintenance policy.

The increasing competitiveness of current markets has resulted in the development of integrated models to exercise better control over the point of view of quality and production performance. For instance, in the paper of Colledani and Tolio [13], it was considered the impact of the quality control action on the logistic flow of parts, where the behavior of the production system was monitored by statistical control charts. Their analytical method was used for evaluating the performance of a production system, in special the system throughput and the system yield. Mhada et al. [14] developed an analytical method for the production control problem for the case where the machine in the operational state systematically produces a fraction of defective parts. They observed that the presence of defective parts, produces a reduction in the effective maximum production rate of the machine. In the paper of Dhouib et al. [15], the authors proposed a mathematical model for the joint determination of production and maintenance policy. Maintenance actions are planned when the system switches to the out of control state and starts producing non-conforming units. In their model, maintenance reduces the shift rate to the out of control state. Hlioui et al. [16] dealt with the coordination of production, replenishment and inspection decisions. Their model determined the ordering point and lot size of raw material, the level of product inventory and the severity of the sampling plan. Paraschos et al. [17] proposed a production and maintenance model for a production system that is affected by frequent deterioration failures. They assumed that the quality of the products is affected by the level of deterioration and maintenance is conducted to prevent any further degradation and keep the system functional. From the above papers, the production-quality interaction has been extensively studied and has provided interesting results. However, there is still a vast number of aspects that has not been included on such models. For instance, the consideration of maintenance policies in the joint production and quality control needs further investigation.

There exists a vast amount of literature on the coupling of production and maintenance planning, since it has been proven that these two key functions are strongly linked. For instance, Yedes et al. [18] presented a model for a production unit that randomly shifts from an in-control to an out of control state, where at the end of each production cycle, corrective or preventive maintenance action is performed, depending on the state of the production unit. In the paper of Hajej et al. [19], it was analyzed a jointly optimal production plan and preventive maintenance program based on an industrial case, where a random product demand must be satisfied with a given required service level. Additionally, the portion of products that are non-conformal are collected and then remanufactured by a subcontractor. Nourelfath et al. [20] studied the problem of integrating imperfect preventive maintenance and production planning in the context of an imperfect process with defective production. In their model, during each period the machine is inspected and imperfect preventive maintenance is performed to reduce its age proportional to the preventive maintenance level conducted. A mathematical model has been developed in Polotski et al. [21] who addressed the problem of joined production and maintenance policy. They considered that the system is capable of sharing its production time between manufacturing mode and remanufacturing mode in which returned products are used in production. Glawar et al. [32] studied a conceptual design of the link between production planning and maintenance modeling. Their design was aimed to reduce the complexity of the planning process and achieve higher levels of availability and efficiency in the resources. They focused in several aspects of data-driven maintenance strategies and autonomous production. As can be noted from the discussed papers, they have hitherto studied the interaction between production and maintenance from different perspectives. Nevertheless, they have ignored the impact of quality issues on such interactions. Thus, we considered in this paper simultaneously the three keys functions of production planning, quality control and maintenance strategies in an integrated model.

Far from being exhaustive, our treatment of the topic covers several models that were suitably adapted to jointly determine quality, production and maintenance planning. Mutual relations among these three key factors should not be underestimated while configuring and managing production systems as a whole. For instance, in the paper of Bouslah et al. [22], it was jointly optimized the production lot size, the inventory threshold, the sampling plan parameters and the overhaul threshold. In their model the quality control is performed using a single acceptance sampling plan by attributes, while overhaul is undertaken once the proportion of defectives reaches a given threshold. Lopes [23] integrated a joint optimization of quality control, buffer and maintenance scheduling for an imperfect production system. In which a percentage of the parts produced are inspected, also in their model they assumed that the inspection policy is imperfect. In the paper of Rivera-Gómez et al. [24], it was analyzed the optimal production and repair/major maintenance switching strategy for an unreliable manufacturing system, where the effects of the wear of the production unit were mainly observed on the failure intensity and on the quality of the parts produced. Unfortunately, in their model they did not include quality control policies, they only model quality deterioration. Recently, Rivera-Gómez et al. [25] presented a simulation-based optimization approach for the joint control of production, preventive maintenance and quality sampling plan, characterized by a priori knowledge of the structure of the control policies adapted from the literature. Hence, their model lead to suboptimal solutions. They considered in their model quality deterioration and an outgoing quality constraint and did not consider that deterioration may have effects on the system reliability. Abubakar et al. [26] proposed a joint control of production, quality and maintenance of a production system. They used a statistical process control to supervise the system and the quality of the units. Also they considered the effect of the production rate on the system degradation. Recently, Hajej et al. [27], proposed a production and maintenance strategy taking into account quality deterioration. Also they proposed a dynamic sampling policy which considered the deterioration of the failure rate to adjust the level of inspection. They replaced their stochastic model with a deterministic equivalent model that were solved through numerical procedures. Additionally predictive maintenance, PdM, has attracted attention in recent years to predict failures before they occur [28,29]. However such domain has only suggested the product-quality prediction [30]. The link between production-quality-maintenance has been disregarded. Moreover, companies must consider the trade-off between costs and benefits before adopting PdM, because in some cases it is more convenient to use a preventive maintenance approach rather than PdM due to the investments in IT infrastructure, the cost of data science professionals, etc., [31].

Summing up, Table 1 highlights the contribution of the paper. The categories I–V, classify the discussed papers according to the research areas that have spurred significant contributions to the domain, while the columns of Table 1 identify key factors of such papers. These factors are the hallmark of current literature.

In Table 1, we identify the research opportunities that are addressed in this paper. The contribution of the paper seeks to develop a new integrated model considering production, dynamic sampling inspection and maintenance policies for an unreliable system subject to quality and reliability deterioration. We aim to jointly optimize these interrelated policies minimizing the total incurred cost and satisfying an outgoing quality constraint. The focus of the paper is to extend existing literature, mainly the papers of Rivera-Gómez et al. [25] and Bouslah et al. [22,33] by developing an integrated stochastic optimal control model where quality decisions are not dissociated from production and maintenance planning. With the proposed model, we aim to show that a dynamic sampling plan can lead to significant economic savings compared with current models. Furthermore, the production strategy is also adjusted according to the deterioration level and major maintenance can be conducted during production to mitigate the effects of such deterioration process. As noted in Table 1, and to the best of our knowledge prior work has not jointly addressed the set of characteristics studied in this paper. Further, an optimization model based on stochastic dynamic programming approach is developed to determine the structure of the joint control policies. Additionally, numerical examples and an extensive sensitivity analysis are conducted to explore the effects of several cost and system's parameters on the optimal control policy.

The rest of the paper is organized as follows. Section 2 presents the industrial context that motivated the research, Section 3 introduces the notations and the problem under consideration. Section 4 is devoted to the detailed formulation of the related stochastic control model under analysis. Section 5 focuses on the development of a numerical example and the determination of the control policy. An extensive sensitivity analysis is conducted in Section 6. A comparative study is presented in Section 7 where the economic benefits of the proposed approach are highlighted. Finally, Section 8 concludes the paper and presents some directions for future research.

2. Industrial context

The main economic and industrial impact of this research lies in the fact that in several real production systems there is the current need of addressing the negative effects of deterioration through effective production management policies. For instance, it is common to observe the adverse effects of deterioration phenomenon in various key manufacturing sectors such as electronics, chemical companies, aeronautic, automobile, machining [7]. However, the field of manufacturing systems has disregarded the impact of deterioration on the determination of production management and maintenance policies, and mainly it has been disregarded the impact on the quality control policy. The economic viability of the developed model relies in the current industrial need of addressing the effects of degrading processes with continuous deterioration of part quality [13]. Additionally, production systems composed with a large number of parts such as machining centers, CNC lathes, mills, progressively deteriorate, having a strong impact on the product quality. Nevertheless, quality deterioration is commonly ignored in production-quality-maintenance policies. Therefore, addressing this gap, certainly leads to economic benefits to the company due to the reduction in the total cost.

The presence of quality deterioration imposes a challenge to efficiently control manufacturing systems. However, a number of quality control policies are based on static sampling plans that disregards the strong interactions with production and maintenance strategies [33]. This traditional quality control (such as ANSI/ASQC Z1.4 and ISO 2859) is not reliable since several manufacturing processes such as automobile, electronics, chemicals, etc. are affected by progressive deterioration [5, 6]. In this context, we aim the fill these gaps in the literature at proposing an innovative integrated model considering a dynamic sampling strategy and also analyze their interactions with production and maintenance planning. The developed model is suitable for deteriorating production systems that are subject to random failures, where their production, maintenance, and inspection rates are controlled.

3. Notations, problem description and assumptions

This section presents the notations used throughout this article, and the problem description.

3.1. Notations

The problem under consideration is based on the following notations:

$x(t)$: Inventory level at time t
$a(t)$: Age of the machine at time t
$\xi(t)$: Stochastic process at time t
d : Demand rate of products
τ_p : Production time
τ_c : Quality control time
u : Production-Quality rate
u_p : Production rate
u_c : Quality control rate
u_{max} : Maximum production rate
$Q(\cdot)$: Transition rate matrix of the stochastic process
ρ : Discount rate
π_i : Limiting probability at mode i
$\lambda_{\alpha\alpha'}(\cdot)$: Transition rate from mode α to mode α'
$g(\cdot)$: Cost rate function
$J(\cdot)$: Expected discounted cost function

Table 1
Summary of contribution of different authors.

	Production policy	Quality control policy	Maintenance policy	Quality level constraint	Sampling inspection	Dynamic sampling	Dynamic production threshold	Quality deterioration	Failure rate deterioration	Proposal of new policy	Optimal control
I. Inspection strategies											
Yoo et al. [1]	✓	✓			✓						
Hsu and Hsu [2]	✓	✓			✓			✓			
Chen [3]	✓	✓	✓		✓			✓			
Mhada et al. [4]	✓	✓						✓			✓
Ait-El-Cadi et al. [5,6]	✓		✓		✓			✓	✓		
II. Deterioration											
Kouedeu et al. [7]	✓		✓				✓		✓	✓	✓
Ayed et al. [8]	✓		✓						✓		
Oosterom et al. [9]			✓						✓		
Ouaret et al. [10]	✓		✓				✓	✓	✓	✓	✓
Ouaret et al. [11]	✓		✓				✓		✓	✓	✓
Wang et al. [12]	✓		✓					✓	✓		
III. Production and quality strategies											
Colledani and Tolio [13]	✓	✓						✓			
Mhada et al. [14]	✓	✓								✓	✓
Dhouib et al. [15]	✓		✓					✓			✓
Hlioui et al. [16]	✓	✓						✓			✓
Paraschos et al. [17]	✓	✓	✓					✓	✓		
IV. Production and maintenance strategies											
Yedes et al. [18]	✓		✓					✓			
Hajej et al. [19]	✓		✓						✓		
Nourelfath et al. [20]	✓		✓					✓			
Polotski et al. [21]	✓		✓						✓	✓	✓
Glawar et al. [32]	✓		✓				✓				
V. Integrated strategies of production-quality and maintenance											
Bouslah et al. [22]	✓	✓	✓	✓	✓			✓	✓		✓
Lopes [23]	✓	✓	✓	✓	✓			✓			
Rivera-Gómez et al. [24]	✓	✓	✓				✓	✓	✓	✓	✓
Rivera-Gómez et al. [25]	✓	✓	✓	✓	✓	✓	✓	✓			
Abubakar et al. [26]	✓	✓	✓	✓				✓	✓		
Hajej et al. [27]	✓	✓	✓	✓	✓	✓		✓	✓		
The proposed model	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

$v(\cdot)$: Value function
τ : Jump time of $\xi(t)$
c^{+} : Inventory holding cost per unit of produced parts
c^{-} : Backlog cost per unit of produced parts
C_{ins} : Inspection cost
C_{reb} : Scrap cost
C_{def} : Cost of selling-accepting a defective item
C_{rep} : Minimal repair cost
C_{maj} : Major maintenance cost
C_{err} : Cost of error of inspection
$\beta(\cdot)$: Rate of defective items
$f(\cdot)$: Fraction of sampling inspection
θ : Adjustment parameter
$AOQL_{max}$: Average outgoing quality limit required by customers

3.2. Problem description

In this paper, attention is focused on the case of a continuous-flow manufacturing system composed of a single unreliable production unit that supplies a downstream stock of finished products that are used to satisfy product demand as illustrated in Fig. 1. However, the machine is unreliable and is subject to random failures and progressive deterioration that originate severe disruptions in the production system. The deterioration process leads to observe a progressive quality and reliability degradation that implies decreasing product quality and an increasing failure rate. A minimal repair is available to cope random failures leaving the machine in the conditions before failure. Major maintenance can be conducted to restore the machine to initial conditions and mitigate the effects of quality and reliability deterioration. The finished product has one key quality attribute, where if the product does not conform to specifications, it is considered as a defective. After production, a quality sampling plan is conducted to reduce non-conforming product. Both quality control and major maintenance contribute to satisfy quality requirements denoted by the Average Outgoing Quality Limit ($AOQL_{max}$) imposed by customers. We assume that defective parts detected during the inspection are rejected. However, some amount of defectives will reach the final customer, because a 100 % inspection strategy is not implemented and only a fraction of the units produced is inspected. The quality control policy consists on an inspection strategy that must satisfy the AOQ constraint in the context of progressive quality deterioration. Furthermore, given that the outgoing quality of the system is strongly influenced by production management, quality control and maintenance policy, these strategies must be jointly determined to guarantee that the customer quality constraint is satisfied. Therefore, the main objective of this research is to develop a new control policy that jointly manages production, sampling inspection and major maintenance strategies and that seeks to minimize the total incurred cost under an outgoing quality constraint. The optimization problem considers the inventory, backlog, maintenance, production, scrap, defectives, inspection and error of inspection costs and such solution satisfies the outgoing customer quality requirement.

3.3. Assumptions and definitions

The production systems under study is based on the following assumptions and definitions:

1) The raw material is always available for the production unit.
2) The customer demand rate for finished products is constant during the time period considered.
3) The repair conducted at failure is minimal, implying a semi-Markov process, where the level of deterioration of the machine remains in *as-bad-as-old (ABAO) conditions.*
4) The major maintenance is perfect, and after such maintenance the machine is rejuvenated to *as-good-as new (AGAN) conditions.*
5) The inspection activity is not perfect, this activity is subject to errors generating additional costs. Inspection errors are due to a number of factors such as inspector fatigue, inefficient training, inconvenient environmental conditions, etc.
6) Defective items identified in inspections are sorted and discarded from the process as scrap.
7) The customer imposes a limit for the amount of defectives that are detected in shipments of finished products.

We have presented these assumptions in order to facilitate the understanding of the formulation of the proposed stochastic control model, the objective of the paper seeks to extend previous models based on these assumptions.

4. Problem formulation

The problem under study consists of an unreliable manufacturing system producing one-part type subject to deterioration. The mode of the manufacturing system at time t is given by the random variable $\xi(t)$

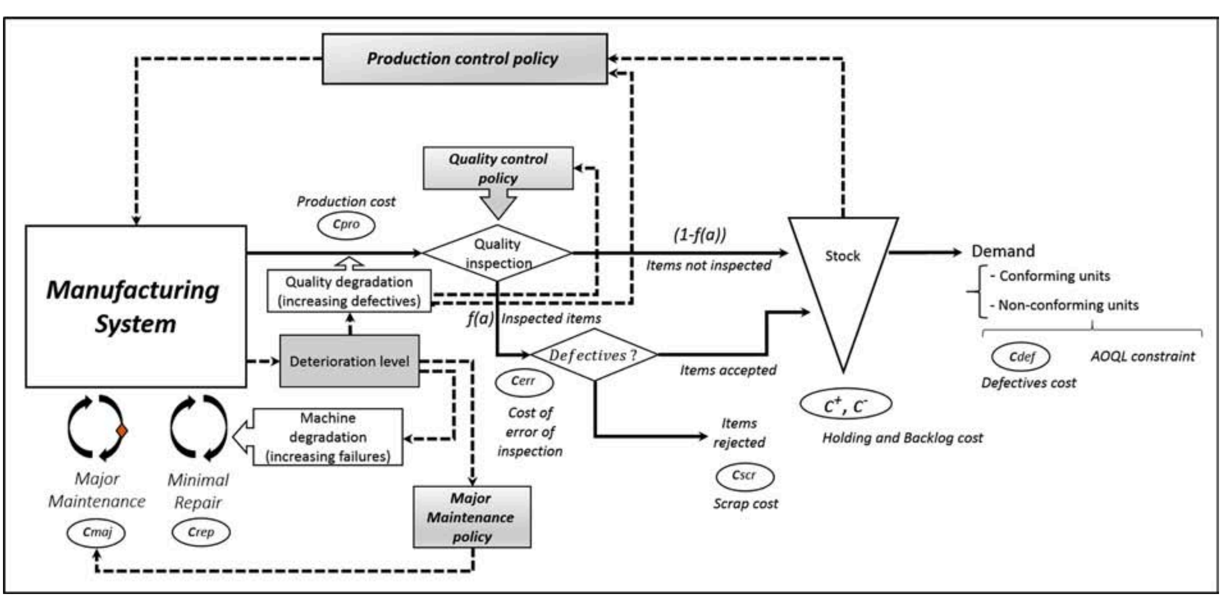


Fig. 1. Production system under study.

with value in $\Omega = \{1,2,3\}$ such that: when $\xi(t) = 1$, the production unit is operational, when $\xi(t) = 2$, the unit is under minimal repair, and when $\xi(t) = 3$, major maintenance is conducted. Then the mode of the production unit at time t is given by the random process $\xi(t)$ as follows:

$$\xi(t) = \begin{cases} 1 & \textit{operational} \\ 2 & \textit{under minimal repair} \\ 3 & \textit{under major maintenance} \end{cases} \tag{1}$$

The transition diagram of such stochastic process is illustrated in Fig. 2.

In our model, $\lambda_{\alpha\alpha'}$ denotes the transition rate from mode $\alpha \in \Omega$ to mode $\alpha' \in \Omega$. The stochastic process is described by the matrix $Q(\cdot) = [\lambda_{\alpha\alpha'}]$ where $\lambda_{\alpha\alpha'}$ verify the following conditions:

$$\lambda_{\alpha\alpha'} \geq 0 \ (\alpha \neq \alpha') \tag{2}$$

$$\lambda_{\alpha\alpha} = -\sum_{\alpha' \neq \alpha} \lambda_{\alpha\alpha'} \tag{3}$$

The transition probabilities of the production unit are given by:

$$P[\xi(t+\delta t) = \alpha' | \xi(t) = \alpha] = \lambda_{\alpha\alpha'(\cdot)}\delta t + o(x, a, \delta t) \tag{4}$$

$$P[\xi(t+\delta t) = \alpha | \xi(t) = \alpha] = 1 + \lambda_{\alpha\alpha(\cdot)}\delta t + o(x, a, \delta t) \tag{5}$$

where $o(x, a, \delta t)$ is a quantity such that

$$\lim_{\delta t \to 0} \frac{o(x, a, \delta t)}{\delta t} = 0 \text{ for all } \alpha,\ \alpha' \in \ \Omega: \ \alpha \neq \alpha'$$

We consider that the age of the production unit at time t is an increasing function of its production rate. Thus, the age of the unit is described by the following differential equation:

$$\frac{da(t)}{dt} = k_1 u(t) \tag{6}$$

$$a(\cdot) = 0 \tag{7}$$

Where $a(\cdot)$ refers to the age of the machine after each major maintenance and k_1 is a positive constant. The transition matrix $Q(\cdot)$ of the stochastic process $\xi(t)$ depends on the age of the machine and the major maintenance rate as presented in Eq. (8). To cope with the effects of deterioration we introduce a control variable $\omega(\cdot) \in \{\omega_{min}, \omega_{max}\}$, where $\omega(\cdot)$ is set to its maximum value ω_{max} if major maintenance is conducted and to ω_{min} otherwise. Furthermore, the transition rate to major maintenance $\lambda_{13}(\cdot)$ can be controlled, since it is a linear function of $\omega(\cdot)$ such as $\lambda_{13}(\cdot) = \omega(\cdot)$. The inverse $1/\lambda_{13}(\cdot)$ represents the expected delay to conduct major maintenance.

$$Q(a, w) = \begin{bmatrix} \lambda_{11} & \lambda_{12}(a) & \lambda_{13}(\cdot) \\ \lambda_{21} & \lambda_{22} & 0 \\ \lambda_{31} & 0 & \lambda_{33} \end{bmatrix} \tag{8}$$

A key concept at the heart of the model is that we assume that the production unit deteriorates with age and we state that such deterioration process has an impact not only on the failure rate but also on the quality of the units produced, as in Kim and Gershwin [39]. Thus the failure rate $\lambda_{12}(a)$ is an increasing function of the age a (t) and it is defined as follows:

$$\lambda_{12}(a) = \eta_0 + \eta_1\left(1 - e^{-\eta_2 \cdot}\left[a(t)^3\right]\right) \tag{9}$$

where parameters η_0, η_1 and η_2 are given constants. We present in Fig. 3 the trend of the failure rate for different values of the parameter η_2. As can be noted in Fig. 3 there is a significant influence of the deterioration of the machine on the failure rate, since λ_{12} increases considerably as the machine ages.

The effect of the deterioration process on product quality has been successfully modeled thorough the used of increasing functions as in Dehayem-Nodem et al. [34]. In this case we model quality deterioration with the following function:

$$\beta(a) = \nu_0 + \nu_1\left(1 - e^{-\nu_2 \cdot}\left[a(t)^3\right]\right) \tag{10}$$

with ν_0, ν_1 and ν_2 defined as given constants. Eq. (10) defines the impact of the level of deterioration of the product quality and serves to represent different quality yields. To illustrate the influence of deterioration on product quality, we present in Fig. 4, the trajectory of the rate of defectives for different values of the parameter ν_2.

It should be noted that Eqs. (9) and (10) serve us to model two different phenomena, namely reliability and quality deterioration. Increasing functions such as Eqs. (9) and (10) illustrated in Figs. 3 and 4, have been frequently used in the domain of unreliable and imperfect manufacturing systems as an effective alternative to model deterioration processes as in Ait-El-Cadi et al. [5,6] and references therein. Furthermore, historical data of maintenance service data and quality is the source to determine the appropriate value of constants η_0, η_1, η_2, ν_0, ν_1 and ν_2 to adjust Eqs. (9) and (10) to a particular production system. Such constants can be determined from this data through estimation methods such as the maximum likelihood and least square.

In the following it is assumed for the sake of simplicity that in the quality control strategy, defective products detected in the inspection are rejected. We use a sampling inspection strategy with the aim to obtain significant economic savings compared to 100 % inspection.

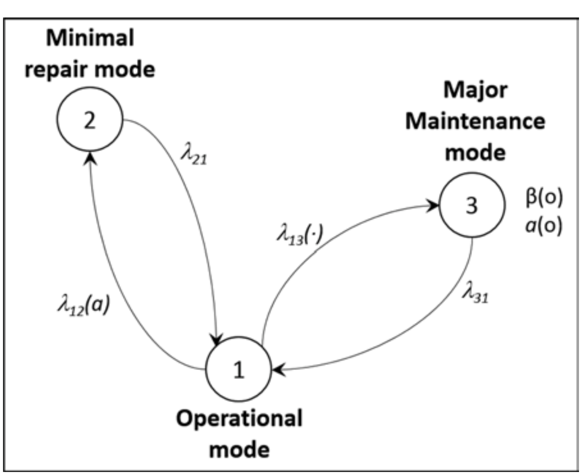


Fig. 2. Transition diagram.

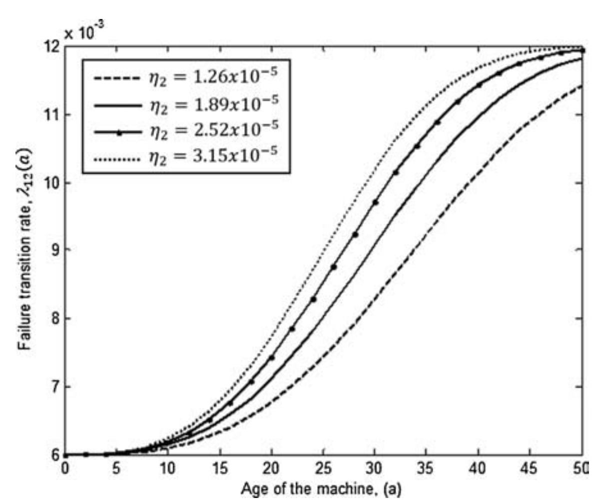


Fig. 3. Trend of deterioration for the failure rate.

However, in several settings, with a sampling plan some products are not inspected; thus defectives will reach the final customer. In this case the long-run average rate of defectives delivered to customers, called the *average outgoing quality* is calculated as follows:

$$AOQ\left(a\right) = \frac{(1 - f(\cdot))\cdot\beta(a)}{1 - f(\cdot)\cdot\beta(a)} \tag{11}$$

where $f(\cdot)$ denotes the fraction of inspected products and a is the age of the machine given by Eq. (6). The indicator $AOQ(a)$ is continuously updated in the model and serves us to define the quality constraint required by customers. The production time τ_p is defined by the inverse of the production rate u_p:

$$\tau_p = \frac{1}{u_p} \tag{12}$$

Where u_p denotes the production rate. Additionally, if we consider that the duration of quality control activities is not negligible, the production capacity of the unit is affected by the quality control delay which mainly depends on the fraction of units that are being inspected. Thus, in this case, the duration of quality control is defined by the following expression:

$$\tau_c = \frac{f(\cdot)}{u_c} \tag{13}$$

where u_c is the quality control rate. A key aspect of Eq. (13), is that as $f(\cdot)$ increases, then more products are inspected, and this has the consequence that more time is needed for quality control. Hence the total production-quality rate u, reduces when more inspection is conducted, as defined in the following expression:

$$u = \frac{1}{\tau_p + \tau_c} \tag{14}$$

Note that the production-quality rate u denoted in Eq. (14) takes into account the delay of quality control activities. Since as the machine deteriorates, then more inspection is conducted and this has the consequence of reducing its production capacity. More formally, the dynamics of the stock is defined by the following differential equation:

$$\frac{dx(t)}{dt} = [1 - f(\cdot)\cdot\beta(a)]\cdot u - \frac{d}{1 - AOQ(a)} \tag{15}$$

where d is the demand rate of the parts produced with the assumption that rejected products by the customer are replaced.

In such a context, the objective of the model is to determine three decision variables, namely, the production rate $u(\cdot)$, the fraction of inspected products $f(\cdot)$ and the rate of major maintenance $\omega(\cdot)$ that minimize the expected total cost. The set of feasibility control policies $\Gamma(\alpha)$, including $u(\cdot)$, $\omega(\cdot)$ and $f(\cdot)$ depends on the stochastic process and is defined as follows:

$$\Gamma(\alpha) = \left\{(u(\cdot), f(\cdot), w(\cdot)\ ) \in \ \mathbb{R}^3, 0 \le u(\cdot) \le \ u_{max}, 0 \le f(\cdot) \le 1,\ \omega_{min} \le \omega(\cdot) \le \omega_{max} AOQ(a) \le AOQL_{max}\right\} \tag{16}$$

Notice that the quality constraint of the model is incorporated in Eq. (16), where $AOQL_{max}$ is the limit for the average outgoing quality required by the final customer. The cost function is given by:

$$\begin{aligned} G(\alpha, x, a, u, f, \omega) = \\ C^{+}x^{+}(t) + C^{-}x^{-}(t) \\ +C_{rep}\cdot Ind\{\alpha = 2\} + C_{maj}\cdot Ind\{\alpha = 3\} \\ +C^{o} with\ \alpha \in \Omega \end{aligned} \tag{17}$$

$$Ind\{\alpha\} = \begin{cases} 1 & if\ \xi(t) = \alpha \\ 0 & otherwise \end{cases} \tag{18}$$

with $x^{+} = max(0, x)$, $x^{-} = max(-x, 0)$, where the constants c^{+} and c^{-}, serve us to penalize the inventory and backlog, respectively. The minimal repair cost is denoted by c_{rep} and the major maintenance cost is c_{maj}. The C^{o} cost includes the inspection cost C_{ins}, the cost of scrap C_{reb}, the cost of selling a defective item that was not inspected C_{def}, the cost of production C_{pro}, and the cost originated by the error of inspection C_{err} as denoted in Eq. (19).

$$\begin{aligned} C^{o} = & \\ & C_{ins}\cdot[u(\cdot)\cdot f(a)] && \text{(inspection cost)} \\ & +C_{reb}\cdot[u(\cdot)\cdot f(a)\cdot\beta(a)] && \text{(scrap cost)} \\ & +C_{def}\cdot[\mathrm{d}\cdot AOQ(a)] && \text{(cost of selling a defective item)} \\ & +C_{pro}\cdot u(\cdot) && \text{(cost of production)} \\ & +\frac{f\cdot C_{err}}{(1 - \alpha_1 f(\cdot))^2} && \text{(cost of error of inspection)} \end{aligned} \tag{19}$$

With $0 \le \alpha_1 < 1$, where historical data of quality service is the source to determine the appropriate value of constant α_1. The cost C_{err} allows us to model the fact that the inspection is not perfect, this activity is subject to errors. When $f(\cdot)$ increases, the inspection error increases due to inspector fatigue. Furthermore, in real production, other factors such as inefficient training for inspectors, inconvenient environmental conditions, etc., affect the inspector and reduce his capacity to identify defects [38]. However, to facilitate matters we use a quadratic function to model the cost of error of inspection. Quadratic functions have been successfully used in production models as noted in Nahmias and Olsen

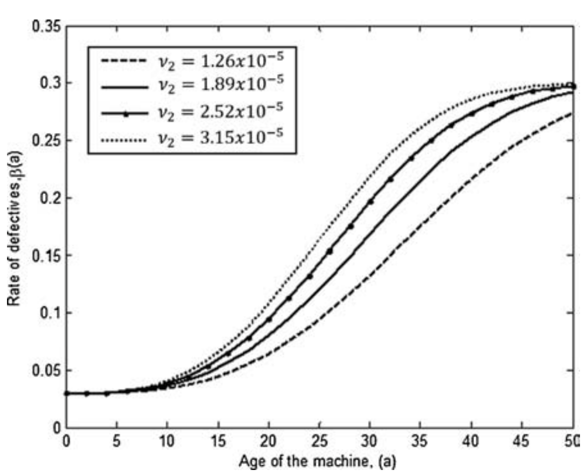


Fig. 4. Trend of deterioration for the rate of defectives.

[35]. In particular, we model inspection errors with the expression $f\cdot C_{err}/(1-\alpha_1 f(\cdot))^2$ where it is apparent to note that when $f(\cdot)\to 1$, more units are inspected. However more errors are done during inspection because more defectives are not properly identified and this increases the incurred cost. Furthermore when $f(\cdot)\to 0$, the error cost $\to 0$. The consideration of the error of inspection serves us to model more realistic quality control policies. The objective of the model is to find in $\Gamma(\alpha)$ a control policy (u^*, f^*, ω^*) which minimizes the following value function:

$$V(\alpha,x,a) = \min_{(u,f,\omega)\in\Gamma(\alpha)} E\left[\int_0^\infty e^{-\rho t}G(\cdot)dt \middle| \begin{matrix}\xi(0)=\alpha,\\ x(0)=x,\\ a(0)=a\end{matrix}\right] \tag{20}$$

where ρ is a given positive discount rate and $V(\cdot)$ is the value function of the problem. Optimality conditions of this problem, can be found by using a stochastic dynamic programming approach. In this case the determination of the optimal controls (u^*, f^*, ω^*) are based on the state variables (α, x, a), where α is the discrete component that defines the mode of the machine and (x, a) are continuous variables that define the stock level and the age of the machine.

The properties of the value function $V(\cdot)$ and the details to obtain the optimality conditions can be found in Appendix A. We replace $\frac{dx}{dt}$ and $\frac{da}{dt}$ by Eqs. (15) and (6), respectively, in Eq. (A.7) to obtain the following Hamilton-Jacobi-Bellman equations:

$$\rho V\left(\alpha,x,a\right) = \min_{(u,f,\omega)\in\Gamma(\alpha)}\left[\begin{matrix} G(\cdot)+\frac{\partial V}{\partial x}(\cdot)\left([1-f(\cdot)\cdot\beta(a)]\cdot u-\frac{d}{1-AOQ(a)}\right)\\ +\frac{\partial V}{\partial a}(\cdot)\left(k_1\cdot u_p(t)\right)\\ +Q(\cdot)V(\alpha,x,\varphi(\xi,a))(\alpha)\end{matrix}\right] \tag{21}$$

where $\frac{\partial V}{\partial x}$ and $\frac{\partial V}{\partial a}$ denote the partial derivatives of the value function. From a mathematical point of view, closed-form solutions of Eq. (21) is a challenge considered unsurmountable, mainly due to the quality constraint of the problem $AOQ(a) \le AOQL_{max}$, the stochastic process $\xi(t)$, and the effects of deterioration.

Given the complexity in solving the HJB type equations described by Eq. (21) it is widely known that an analytical solution of such equations is unsurmountable. However, there is an alternative to perform an approximation of the solution using numerical techniques based on the Kushner approach [36]. This approach has been successfully applied to solve this kind of problems as in Kouedeu et al. [7] among others. The Kushner technique is based on finite difference approximations of the partial derivative of the value function. In particular, it approximates the gradient of the continuous value function $V(\cdot)$ with a discrete function $V_h(\cdot)$. Following the works of Ouaret et al. [10], we refer the reader to Appendix B to get an insight on the numerical approach used to obtain the discrete version of the optimality conditions given by Eq. (B.6). Then the obtained discrete HJB equations can be solved using policy improvement and successive approximation methods.

5. Numerical example and optimal control policy

In the sequel, we will restrict the discussion to the performance of a numerical example to study the strong interactions between production management, quality control and maintenance planning. Problems of this type have a quite complex mathematical structure; thus, such problems are generally tackled by solving the discrete version of Eq. (21) as presented in Appendix B. A finite grid is needed to define the computational domain for the state variables (x, a) as follows:

$$G_{xa} = \{(x,a):\ -20 \le x \le 140,\ 0 \le a \le 50\ \} \tag{22}$$

The ranges denoted in Expression (22) are needed in the numerical technique. The limiting probabilities of modes $\xi(t) \in \Omega$, (i.e. π_1, π_2 and π_3) can be calculated as follows:

$$\begin{cases}\pi_i\cdot Q(\cdot)=0\\ \sum_{i=1}^{3}\pi_i = 1\end{cases} \tag{23}$$

where $Q(\cdot)$ is the transition rate matrix given by Eq. (8). For the sake of conciseness, we must ensure that the production system is able to satisfy customer's demand in cases of high deterioration. Thus, the production system must satisfy the feasibility condition:

$$\pi_1\cdot u_{\max} \ge \frac{d}{1-AOQ(a)} \tag{24}$$

where π_1 is de limiting probability at the operational mode. The parameters used in the numerical example are presented in Table 2 and they satisfy condition (24).

The value of the cost parameters of the numerical example are presented in Table 3.

5.1. Production policy

In this section we define the procedure to determine appropriate decisions for the first decision variable $u^*(\cdot)$ of the proposed model. In this case as previously stated in Section 4, such variable $u^*(\cdot)$ is related to the production control policy. In this regard after applying the Kushner's approach as indicated in Appendix B, we obtained that the optimal production control policy suggests that the effects of random and more frequent failures and large quantity of defective items are mitigated by increasing the stock threshold as presented in Fig. 5. The computational domain is divided in three regions where the production policy consists of the following rules:

1 If the current stock level is less than the production threshold, then the production rate is set to its maximum value.
2 If the current stock level is equal to the production threshold, then the production rate is set to $d/(1-AOQ(a))$.
3 If the current stock level is superior to the production threshold, then the production rate is set to zero.

In what follows, the production control policy is given by the next equation:

$$u^*\left(1,x,a\right) = \begin{cases} u_{max} & \text{if } x(t) < Z_p(a)\\ \frac{d}{(1-AOQ(a))} & \text{if } x(t) = Z_p(a)\\ 0 & \text{Otherwise}\end{cases} \tag{25}$$

where $Z_p(a)$ is the age-dependent function that defines the optimal production threshold in the operational mode as illustrated in the production trace of Fig. 5b. Additionally, from Eq. (25), the production decisions are made based on the value of the state variables (α, x, a). The inventory level $x(t)$ must be continuously monitored to implement the production policy, and the decision to change the production rate is based on the level of stock with respect to the production threshold $Z_p(a)$, (Fig. 5.b).

5.2. Major maintenance policy

The second decision variable of the model, refers to the rate of major

Table 2
Parameters for the numerical example.

u_{max}	d	ρ	h_x	h_a	u_c	λ_{21}	α_1	λ_{31}	$AOQL$
12	6	0.05	5	2	40	0.1	0.9	0.2	0.10
η_0	η_1	η_2	ν_0	ν_1	ν_2	ω_{min}	ω_{max}	k_1	
0.006	0.006	$3.15x10^{-5}$	0.03	0.27	$3.15x10^{-5}$	10^{-6}	500	0.1	

Table 3
Cost parameters for the numerical example.

C^+	C^-	C_{def}	C_{scr}	C_{ins}	C_{pro}	C_{err}	C_{rep}	C_{maj}
10	500	150	5	5	10	25	300	3000

maintenance $\omega^*(\cdot)$. This section is devoted to clearly indicate how major maintenance decisions must be conducted. The major maintenance policy is presented in Fig. 6a, where we notice that the computational domain is divided into two different regions in function of the level of deterioration of the machine. To define the maintenance policy, we need to recall that the inventory level is limited by the production threshold $Z_p(a)$. Thus, considering such limitation, the zone II presented in Fig. 6a, is reduced to the feasible zone II shown in Fig. 6b. The interjection of the maintenance trace and the production threshold defines the critical age A_o.

The trace of Fig. 6b defines the maintenance policy with the help of the following two zones:

Zone I: The conduction of major maintenance is not recommended since the level of deterioration of the machine has no significant effects on its performance and it has enough capacity to satisfy product demand. Hence the decision variable is set to its minimum value $\omega(\cdot) = \omega_{min}$, where $\omega_{min} = 10^{-6}$.

Feasible Zone II: The intersection of Zone II in Fig. 6a and the production threshold $Z_p(a)$ defines the feasible zone II, where the production unit has surpassed the critical age A_o. In such zone the failure rate as well that the defective rate are considerable high, thus the cost of a major maintenance is justified. Additionally, some inventory level is needed to hedge against possible shortages at nonproductive time. In

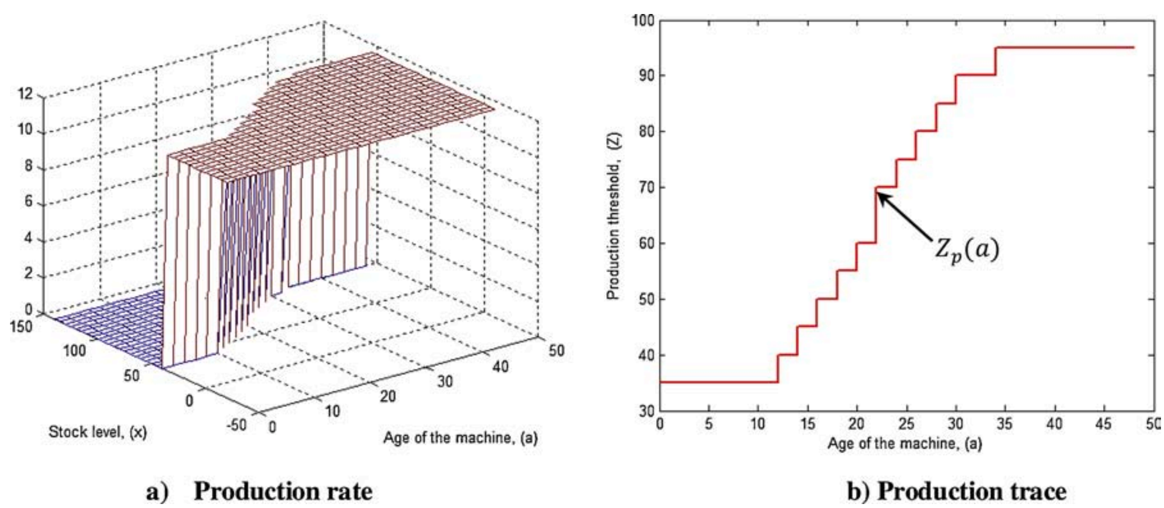


Fig. 5. Production policy.

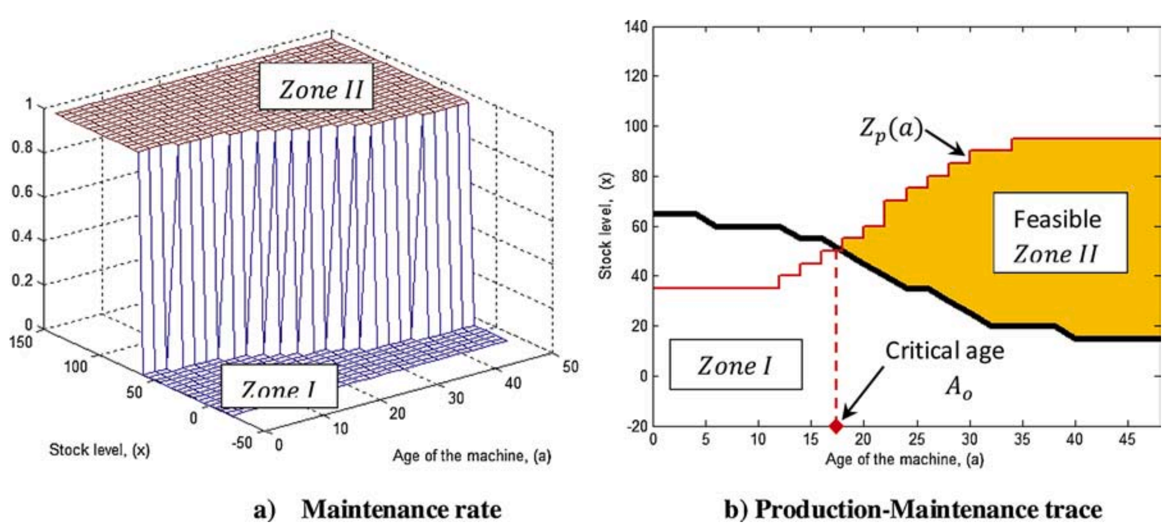


Fig. 6. Maintenance policy.

this case the decision variable is set to its maximum value $\omega(\cdot) = \omega_{max}$, where $\omega_{max} = 500$. We use such value to indicate a negligible delay (i.e., $1/\omega_{max}$ units of time) to conduct major maintenance.

Under these observations, the results of Fig. 6 shows that the optimal control for the major maintenance strategy has a bang-bang policy, where the major maintenance decision variable switches from the lower bound to the upper bound as follows:

$$\omega^*(1, x, a) = \begin{cases} \omega_{max} & \text{if } (x, a) \in \text{Zone II and } a > A_o \\ \omega_{min} & \text{otherwise} \end{cases} \tag{26}$$

where A_o denotes a critical age for major maintenance. As can be noted in Eq. (26), major maintenance decisions are based on two conditions, first major maintenance is triggered if the age of the machine is superior to the critical age A_o, this to justify the high cost of this action. Second, this action is conducted if the state variables (x, a) are in the feasible Zone II, because some amount of inventory is needed to palliate shortages during the period of inactivity and the age must be high enough to justify a major intervention.

5.3. Quality control policy

In this section we define appropriate decisions for the third decision variable of the model related in this case with the fraction of inspected units $f(\cdot)$. In particular, the quality control policy, defines what fraction $f(\cdot)$ of units must be randomly inspected. One technical advantage of our quality control approach is that $f(\cdot)$ is not constant as in Bouslah et al. [33]. The fact that the sampling fraction is a decision variable serves to adjust the level of inspection $f(\cdot)$ in function of the degree of deterioration of the machine. This feature leads to considerable cost saving as reported in the comparative study section (see Section 6).

For the sake of completeness, the obtained quality control policy is presented in Fig. 7a, where we note the progressive increment of the fraction sampling $f(\cdot)$ as the machine deteriorates. Further, to complement the results we present in Fig. 7b the average outgoing quality indice *AOQ*, related to the optimal sampling fraction $f^*(\cdot)$ and rate of defectives $\beta(a)$ as defined in Eq. (10). Fig. 7b serves to illustrate the influence of the $AOQL_{max}$ constraint in the sampling inspection, since we noted that as the quality constraint is stricter ($AOQL_{max}$ reduces) it implies that more products must be inspected, thus this considerably increases the fraction of inspection $f(\cdot)$ to satisfy the customer's quality requirements.

Based on the results of Fig. 7a the quality control policy can be defined as follows:

$$f^*\left(1, x, a\right) = \begin{cases} 0 & \text{if } a(\cdot) \le A_I \\ T_f(a) & \text{if } a(\cdot) > A_I \end{cases} \tag{27}$$

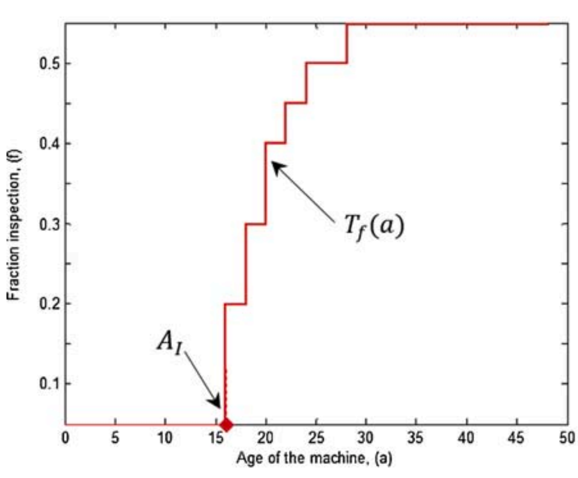
where A_I is the age limit to conduct inspection and $T_f(a)$ is the function that defines the progressive increment on the fraction of inspection. Notice that Eq. (27) indicates how inspection decisions must be made, since inspection must be conducted only when the age of the machine surpasses the critical age A_I, because before such age the machine is in excellent conditions producing a very low amount defectives. Therefore, inspection is not recommended. As the machine age surpasses A_I, the fraction of inspection must be progressively adjusted $T_f(a)$, (see Fig. 7.a) to compensate for the increment of defectives due to deterioration.

6. Sensitivity analysis

The obtained joint control policy is examined through an extensive sensitivity analysis, where we analyze the effect of the variation of some cost and system's parameters on the control policy. Also, we analyze the influenced of the $AOQL_{max}$ constrained required by customers on the joint control policy. The analysis has been conducted using the numerical example of the previous section as base of the study.

6.1. Influence of the $AOQL_{max}$ on quality constraint

We first analyze the case in which the influence of the $AOQL_{max}$ on quality constraint on the control policy is examined. We employ three different values $AOQL_{max} = 0.070$, 0.085 and 0.100 for the analysis. From Fig. 8a we can observe that when small values of $AOQL_{max}$ are required, for instance 0.070 and 0.085, we note that it leads to an increase in the severity of the optimal sampling inspection, thus $f^*(\cdot)$ increases in order to satisfy the $AOQL_{max}$ (quality limit required by customers), and so less defective reaches the final customer. However, when higher values of $AOQL_{max}$ are required, for example at value 0.10, the inspection is less severe, hence the optimal sampling fraction $f^*(\cdot)$ reduces and this has the inconvenience that more defectives reach the customer. In addition, to complement the discussion, Fig. 8b shows the values of the AOQ obtained at implementing the optimal sampling fraction $f^*(\cdot)$ of Fig. 8a. The results clearly indicate that for all the analyzed cases, the value of the $AOQL_{max}$ is always satisfied by the optimal sampling fraction $f^*(\cdot)$ as the machine ages. The variation of the $AOQL_{max}$ constraint reported significant effects on the severity of inspection policy.



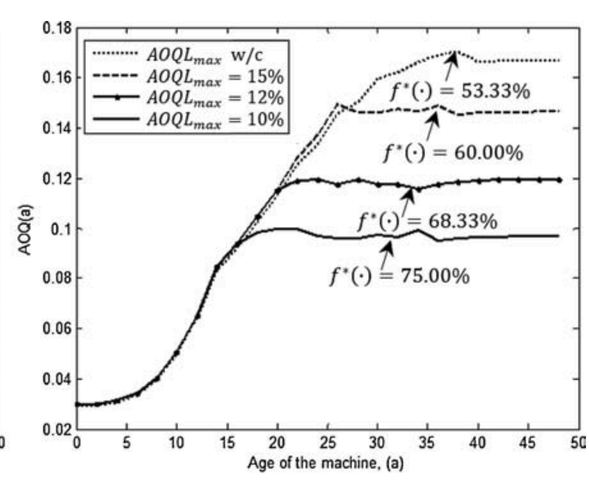


Fig. 7. Quality control policy.

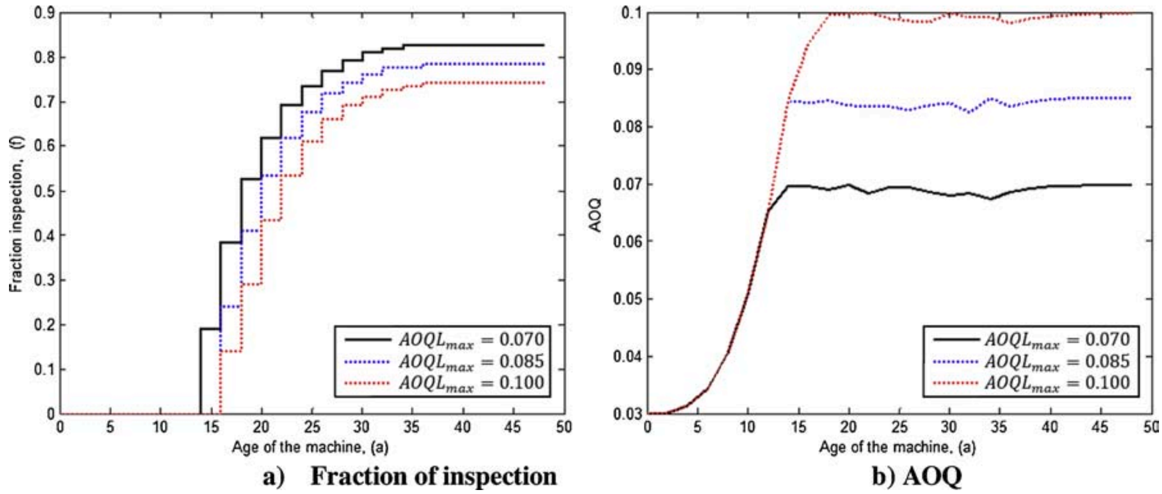


Fig. 8. Effect of the $AOQL_{max}$ constraint on the quality control policy.

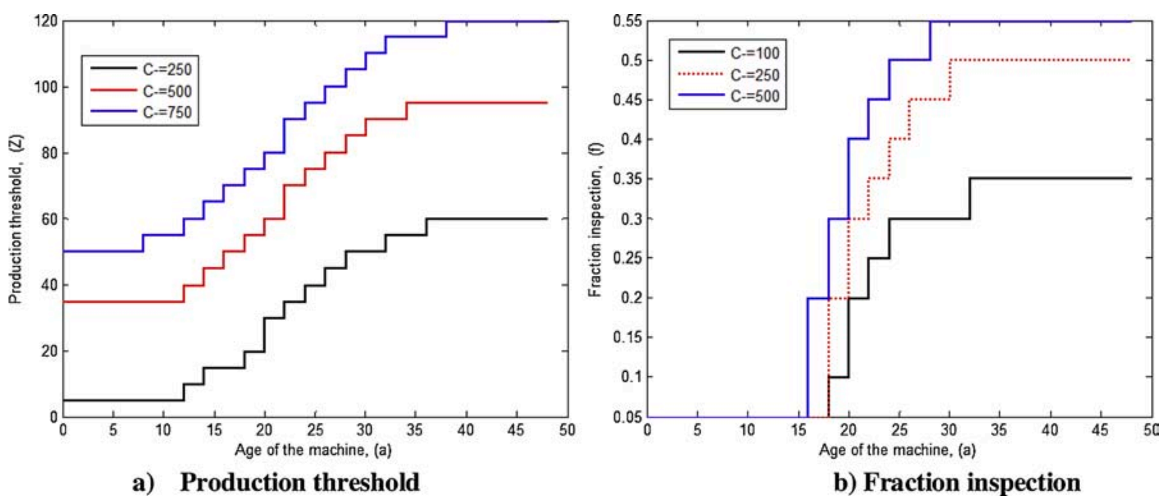


Fig. 9. Effect of the backlog cost on the production and quality control policies.

6.2. *Influence of the cost parameters variation*

In this subsection, the impact of a number of cost parameters is examined in detail. The costs considered in the analysis are the cost of backlog, major maintenance, production, inspection, defectives and error of inspection.

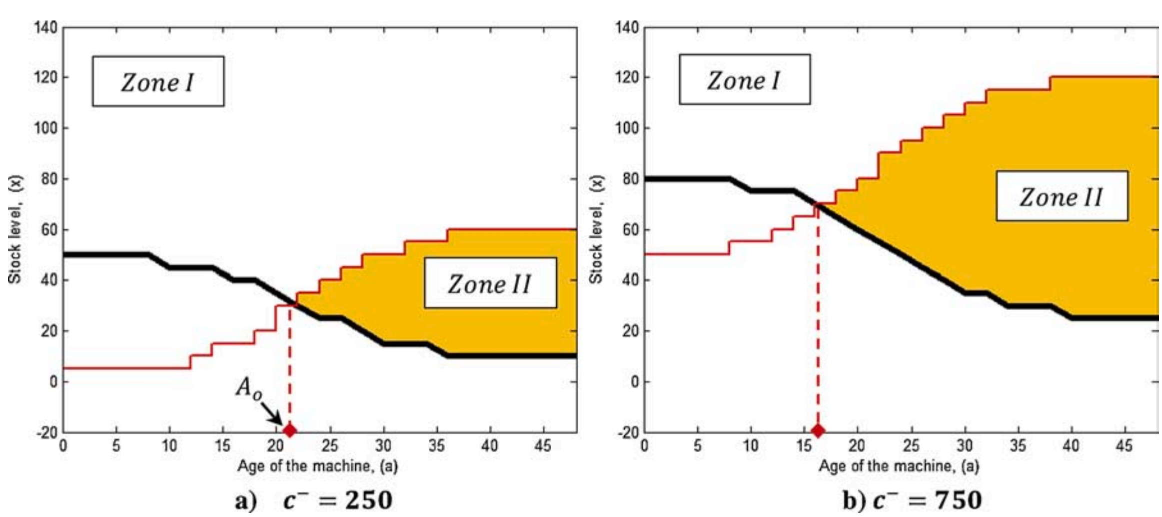


Fig. 10. Effect of the backlog cost on the major maintenance policy.

6.2.1. Variation of the backlog cost

The results presented in Fig. 9a for three different backlog cost values $C^- = 250$, 500 and 700, indicate that when the backlog cost increases the production threshold increases considerably. That is because, the production threshold must be augmented as protection in order to avoid further shortages. Regarding the quality control policy, we note that as C^- increases, the backlog is more severely penalized, thus the inventory level increases, leading that the machine operates more time at its maximum rate. In this context the machine deteriorates faster, generating more defectives, hence more inspection is needed to satisfy the AOQ constraint as C^- increases, as presented in Fig. 9b.

In more details, we can observe in Fig. 10b that with the increment of the backlog cost, more major maintenance is conducted, increasing then zone II, also we notice that the critical age A_o that triggers major maintenance reduces. Then such maintenance is conducted more frequently in order to restore the machine to initial conditions and mitigate all the effects of the deterioration process. Therefore, it is clear that the variation of the backlog cost is directly associated with the size of the major maintenance zone. Regarding the variation of the inventory cost, we noted that it has opposite effects on the control policy that the backlog cost. The analysis for the inventory cost was conducted but for the sake of brevity, is not presented in the paper.

6.2.2. Variation of the major maintenance cost

We illustrate the effect of the variation of the major maintenance cost on Fig. 11 with two different values $C_{maj} = 3000$ and 7500. It is evident in Fig. 11a that when C_{maj} decreases, more major maintenance is recommended and the critical age A_o reduces. The reason to observe this condition is because at increasing C_{maj} it is preferable to postpone the conduction of major maintenance and keep the machine operational for a longer period of time, thus when $C_{maj} = 7500$, A_o increases.

Regarding the effect of the variation of the major maintenance cost on the quality control policy, we note in Fig. 12 an interaction between the cost C_{maj} and the level of inspection. More specifically in Fig. 12 we present the sampling fraction for two different values $C_{maj} = 3000$ and 7500. From the obtained results of Fig. 12 we noted that when the major maintenance cost decreases to $C_{maj} = 3000$, less inspection is recommended. Because more major maintenance is conducted in such case, and the performance of this maintenance options serves as a countermeasure to mitigate the presence of defective units. Conversely, we also note that when the major maintenance increases to $C_{maj} = 7500$, more inspection must be conducted as a mean to palliate the presence of defective units, since with a higher C_{maj}, less major maintenance is conducted.

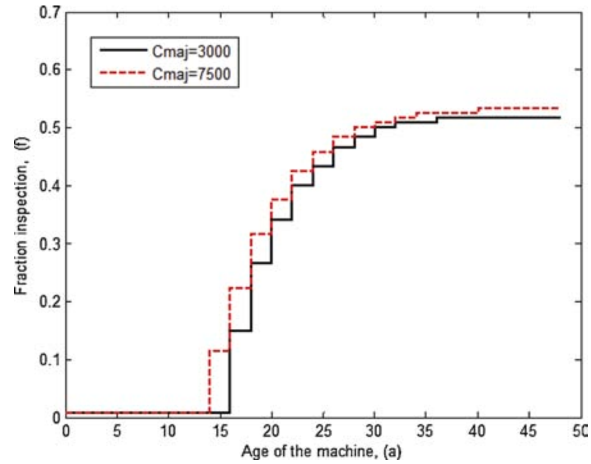


Fig. 12. Effect of the major maintenance cost on the quality control policy.

6.2.3. Variation of the production cost

We analyze the variation of the production cost for three values $C_{pro} = 10$, 100 and 200. The results presented in Fig. 13a indicate that by increasing C_{pro}, the production threshold must be reduced, since the operation of the machine is more penalized. Furthermore, this production threshold reduction implies that the machines is operational less time, thus the machine deteriorates less, which should involve a reduction of the major maintenance zone. However, more inspection is conducted as C_{pro} increases as illustrated in Fig. 13b, to compensate for the reduction of the major maintenance zone.

With respect to the major maintenance policy, we observe that with the increment of the production cost, less major maintenance is conducted as noted in Fig. 14b, because the machine is operational less time when the production thresholds reduce, and this decreases its deterioration rate. As a consequence, the conduction of major maintenance is delayed when C_{pro} increases, and so the critical age rises from $A_o = 17$ to $A_o = 24$. The reduction of the major maintenance zone triggers more inspection.

6.2.4. Variation of the inspection cost

In Fig. 15 we present the effect of the variation of the inspection cost for three different values $C_{ins} = 5$, 30 and 40. In particular, we note that when the inspection cost increases, the optimal sampling fraction decreases reducing then the inspection efforts. Then when we increase C_{ins} it is natural that less inspection is conducted.

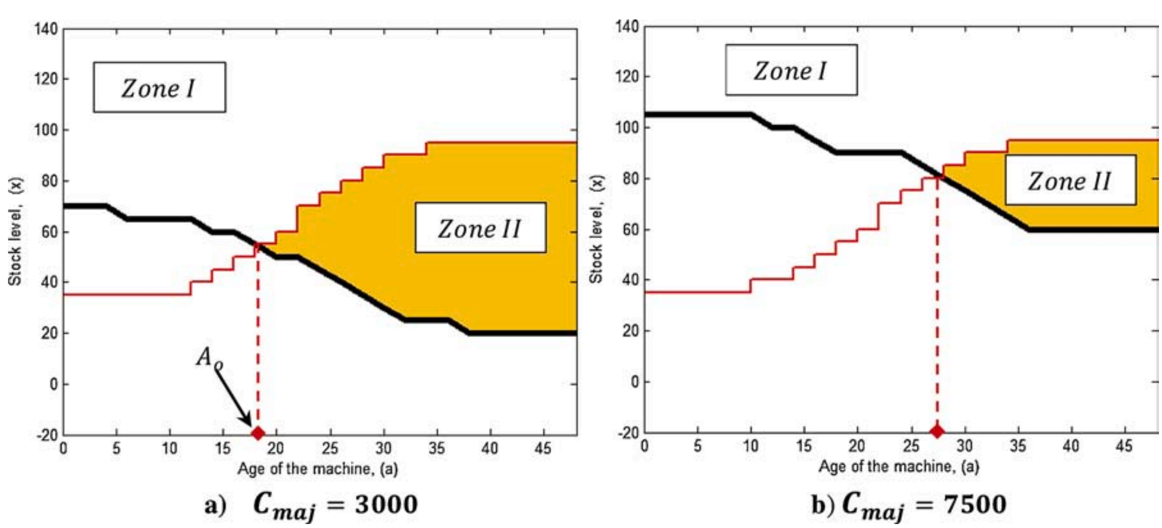


Fig. 11. Effect of the major maintenance cost on the major maintenance policy.

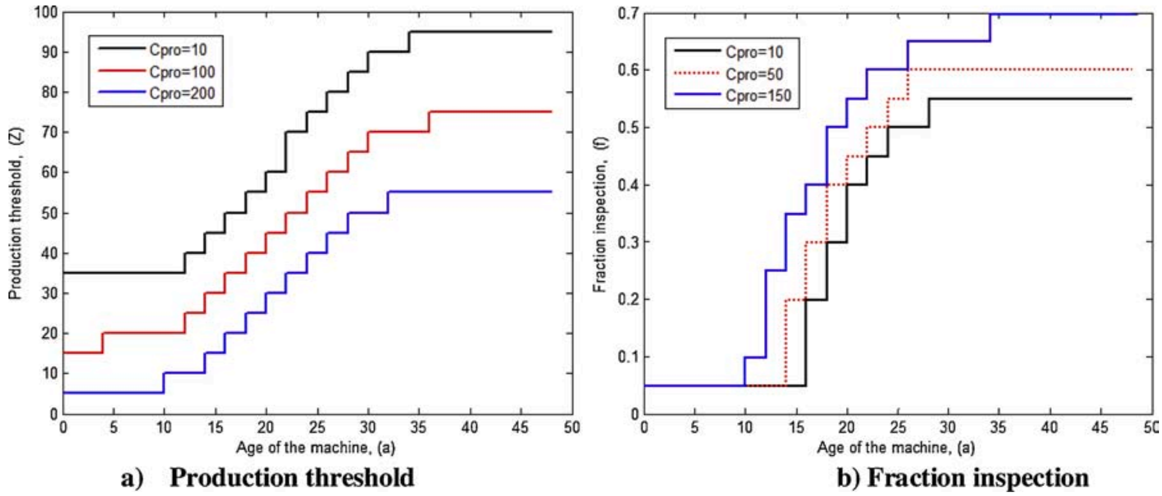


Fig. 13. Effect of the production cost on the production and quality control policies.

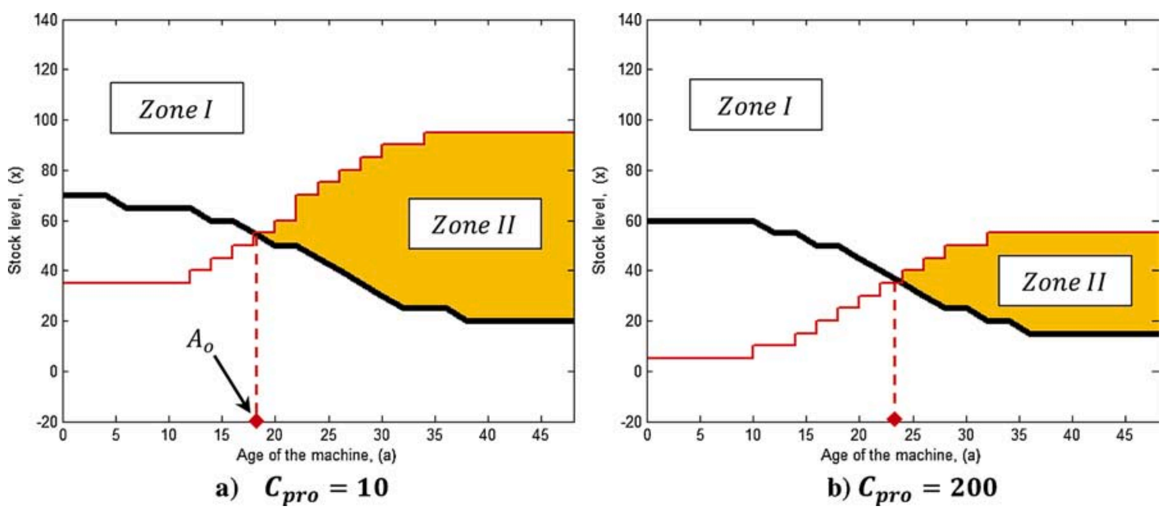


Fig. 14. Effect of the production cost on the major maintenance policy.

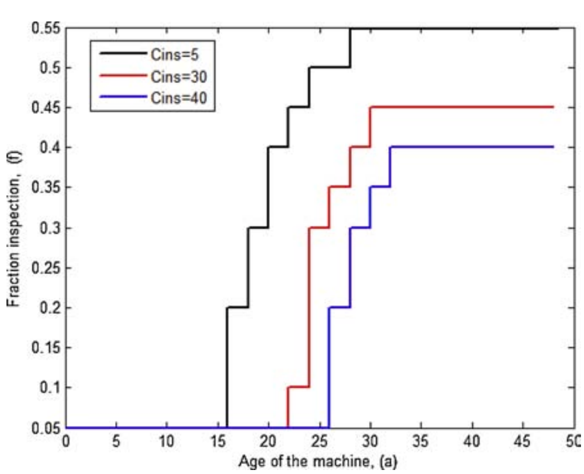


Fig. 15. Effect of the inspection cost on the quality control policy.

We obtained the following interpretation in Fig. 16, we observe an interaction between the inspection cost and the major maintenance strategy. Since when C_{ins} increases, we note that less inspection is conducted. However, more major maintenance is recommended to compensate for the reduction of inspection efforts and mitigate the presence of defective units. The increment of the major maintenance Zone II in Fig. 16b, can be seen as an attempt to reduce the amount of defectives that reaches the final customer. A similar analysis has been conducted for the scrap cost, such results are not presented in the paper, because they reported similar effects on the joint control policy that the inspection cost.

6.2.5. Variation of the defectives cost

We analyze the variation of the cost of defectives for three values C_{def} = 50, 150 and 250. The results presented in Fig. 17 indicate that the defectives cost has a significant effect on the inspection policy. In particular when C_{def} increases, the optimal sampling fraction $f^*(\cdot)$ naturally increases so that the inspection plan becomes more severe, inspecting more units. As C_{def} increases, the company is penalized more rigorously when a defective reaches the customer, thus more inspection is conducted to ensure that customer demand are satisfied with flawless units and avoid further quality costs. It should be noted that a lower cost of C_{def} item has the opposite effects. Further, the defectives cost only reported an effect on the inspection policy.

In this sense, the increment of C_{def} triggers the conduction of more major maintenance, thus zone II increases as noted in Fig. 18b. The conduction of more major maintenance seeks to improve process quality and reduce the total amount of defectives produced. With this

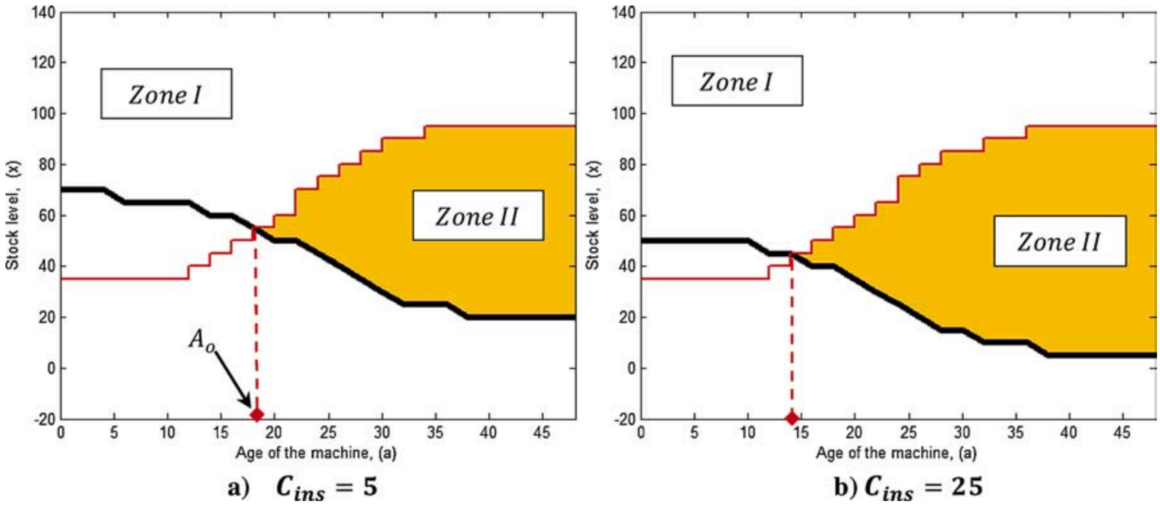


Fig. 16. Effect of the inspection cost on the major maintenance policy.

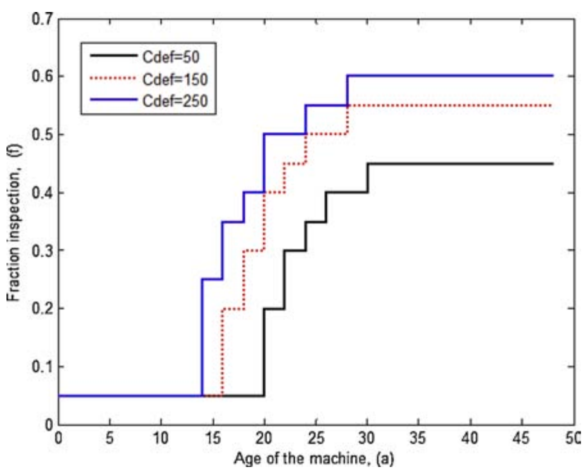


Fig. 17. Effect of the defectives cost on the quality control policy.

countermeasure, due to the conduction of more major maintenance, the system is kept in a better condition, producing less defectives in the long-term. Note that a lower defectives cost produces the opposite effects on the major maintenance and quality control policies.

6.2.6. Variation of the cost of error of inspection

As can be seen from Fig. 19, we use three different cases, $C_{err} = 25$, 40, and 55 to analyze the influence of the cost of error of inspection. We noted that when C_{err} increases, it means that the errors conducted during

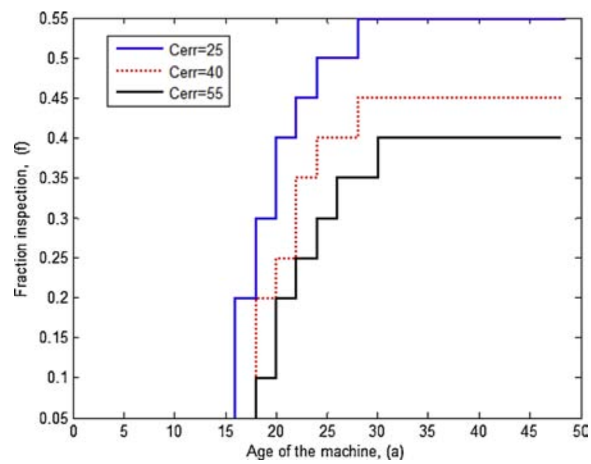


Fig. 19. Effect of the cost of error of inspection on the quality control policy.

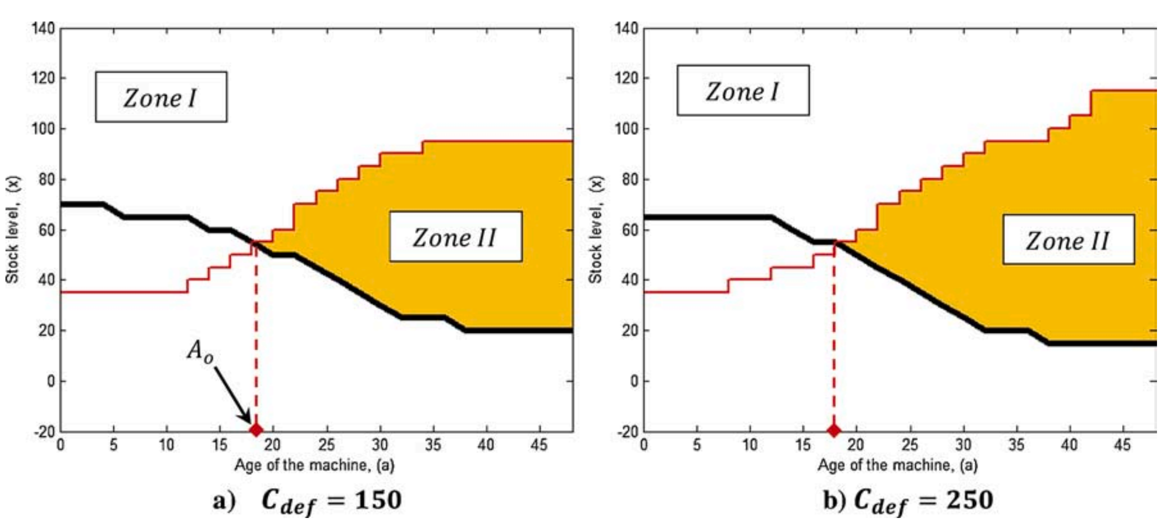


Fig. 18. Effect of the defectives cost on the major maintenance policy.

the inspection are more penalized. Consequently, if C_{err} increases, then the optimal sampling fraction $f^*(\cdot)$ reduces considerably to define the level of inspection to just a necessary level, since the errors conducted in inspection are more penalized. If C_{err} reduces, more inspection is recommended, thus the sampling fraction increases.

Another important aspect regarding the results just presented, concerns that the cost of error of inspection reveals an interaction between the sampling inspection policy and the major maintenance policy. Since we note that when C_{err} increases, less inspection is conducted. Nevertheless, the major maintenance zone increases to enhance the system's condition and ensure that less defectives are being produced and reach the final customer. In the results of Fig. 20b, it is clear the reduction of the critical age A_o when C_{err} increases to 25 and opposite effects are observed when C_{err} decreases.

6.3. Influence of the system's parameters variation

In addition to the previous analysis, we complement the study with the discussion of the variation of two system's parameters, which are related with the trend of the failure rate deterioration and the trend of quality deterioration. We conducted a set of numerical instances to illustrate the influence of these two parameters on the joint control policy.

6.3.1. Variation of the failure rate deterioration

As a matter of interest, in order to clearly illustrate the effect of the variation of the failure rate deterioration, we select the parameter η_2 of Eq. (9). As previously illustrated, such parameter serves us to strongly vary the failure intensity experienced by the production system. To keep things simple, we analyze two different instances as presented in Fig. 21, when the parameter η_2 reduces or increases 25 % from the original value reported in Table 2 (base case). From the obtained results we note in Fig. 20b that when the parameter η_2 increases by 25 %, the production threshold increases, because the machine is less reliable, experiencing more failures. Therefore, more inventory is needed as protection against backlog. Also, we note in Fig. 21b that the increment of η_2 leads to the conduction of more major maintenance, since zone II is defined by the intersection of the production and the major maintenance threshold. Furthermore, we note in Fig. 21a that when η_2 decreases by 25 % the machine is more reliable, experiencing less failures thus less inventory is needed as protection. Further, this threshold reduction leads to decrease the maintenance zone.

A close examination of Fig. 22 indicates that the quality control policy also is affected by the variation of the parameter η_2. In particular, we note that on increasing the parameter η_2 by 25 %, the machine is operational more time because of the increment of the production threshold, thus the machine deteriorates more, producing then more defectives. Therefore, more inspection must be conducted as a countermeasure to palliate the presence of more defective units. When the parameter η_2 decreases by 25 %, less inspection is needed since the machine generates less defectives.

6.3.2. Variation of the quality deterioration rate

One of the factors to be considered in the analysis is the quality deterioration rate. For this purpose, we select the parameter ν_2 to clearly illustrate the effect of the variation of the quality deterioration rate on the joint control policy. The parameter ν_2 allows us to modify the rate of generation of defectives. We analyze and compare two instances where the parameter ν_2 varies by $\pm 20\%$ from the original value reported in Table 2 (base case). In Fig. 23b we note that when ν_2 increases by 20 %, the production threshold increases, because the deterioration rate is accelerated, thus the machine produces more defectives. Therefore, more stock is needed as protection against the presence of defective units. Regarding the major maintenance policy, when ν_2 increases by 20 %, more major maintenance is conducted because the machine generates more defectives. When ν_2 decreases by 20 % of the original value, we note the contrary effects, since the production threshold and the major maintenance zone reduces because the machine generates less defectives.

From a technological point of view, in the case where the deterioration rate is accelerated when v_2 increases by 20 %, we observe in Fig. 24 that more inspection is conducted because when the deterioration rate increases, more defectives are produced. Thus more inspection is needed to satisfy the quality customer's requirements. When v_2 decreases by 20 %, less inspection is recommended since the deterioration rate decelerates, producing less defectives.

7. Comparative study

In the remainder of this section we conduct a comparative study based on the cost representing the value function when the optimum solution is implemented. The comparison is conducted for a given age, where the machine is in a considerable level of deterioration, but remaining capable to satisfy product demand. We use the numerical approach presented in Appendix B to obtain such cost. In the comparison, we called our strategy as Policy-I and also, we consider other policies common in the literature that are based on traditional assumptions. That rely solely on 100 % inspection or the conduction of a fix fraction of inspection for the quality control policy. We also consider policies where there is no preventive maintenance or no quality control. The policies

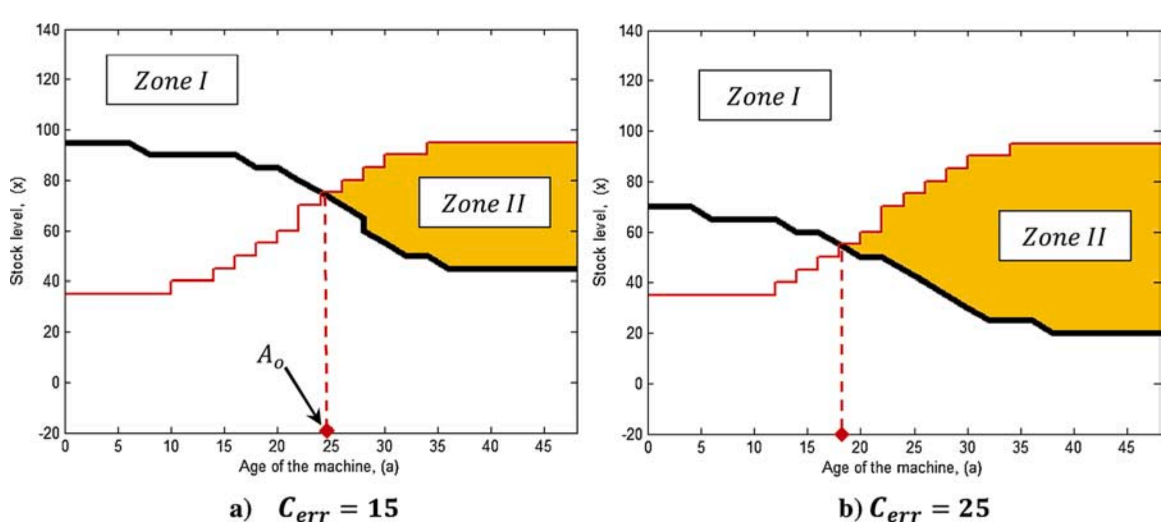


Fig. 20. Effect of the cost of error of inspection on the major maintenance policy.

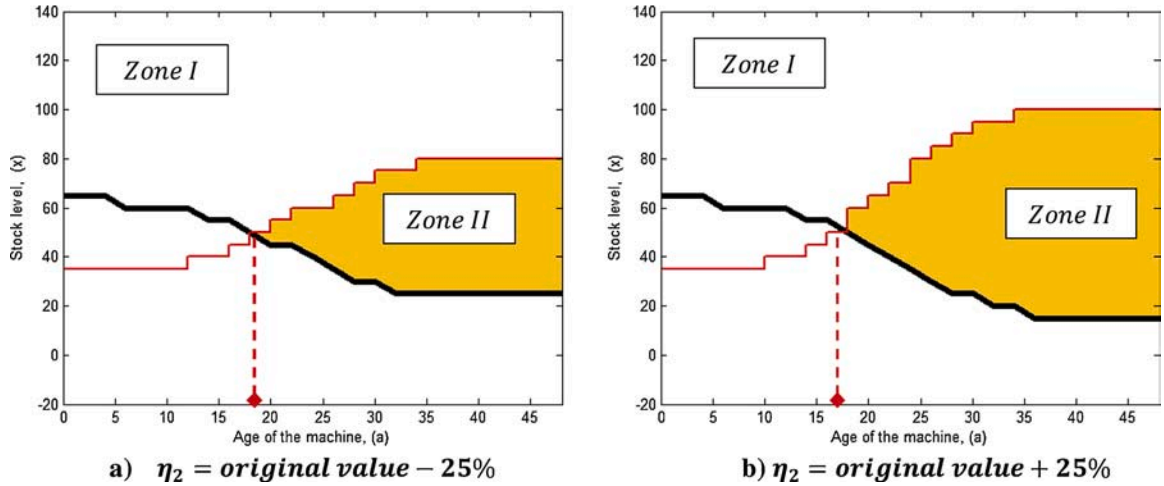


Fig. 21. Effect of the variation of the failure deterioration rate on the major maintenance policy.

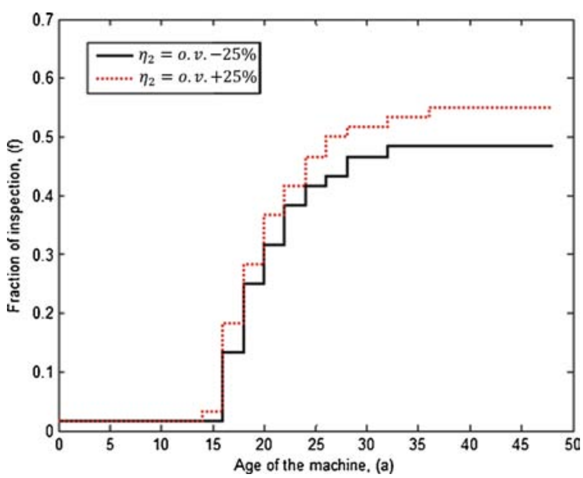


Fig. 22. Effect of the variation of the failure deterioration rate.

considered in the comparative study are defined as follows:

- Policy I: refers to the control policy proposed in this paper. As previously mentioned, the production, inspection and major maintenance rates are jointly optimized through an integrated model. Furthermore, this policy has the distinctive characteristic that the production threshold and the level of inspection are continuously adjusted in function of the level of deterioration of the machine.
- Policy II: in this policy the quality control strategy consists in the classic 100 % inspection regardless of the level of deterioration of the machine. In this policy the production, quality control and maintenance decisions are jointly determined as in Policy-I, but the difference is that Policy-II inspects all the units even in low levels of deterioration.
- Policy III: in this policy the production, quality control and maintenance decisions are determined simultaneously in an integrated model. However, the difference is that in Policy-III the sampling fraction is not dynamic as in Policy-I. In Policy-III we used optimization techniques to determine the most appropriate level of inspection, with the particular feature that the same fraction of units is always inspected, regardless of the level of deterioration of the machine.
- Policy-IV: in this policy the major maintenance strategy is not part of the optimization, the decision variables are only the production rate and the level of inspection which are adjusted according to the level of deterioration of the machine. Hence, we modified Policy-I to exclude maintenance decisions from the optimization.

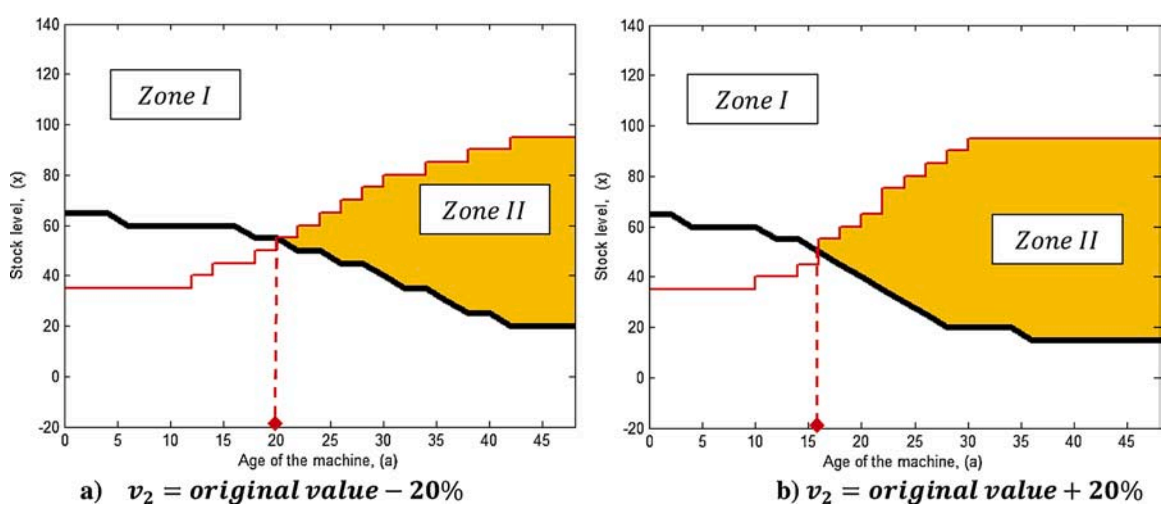


Fig. 23. Effect of the variation of the quality deterioration rate on the major maintenance policy.

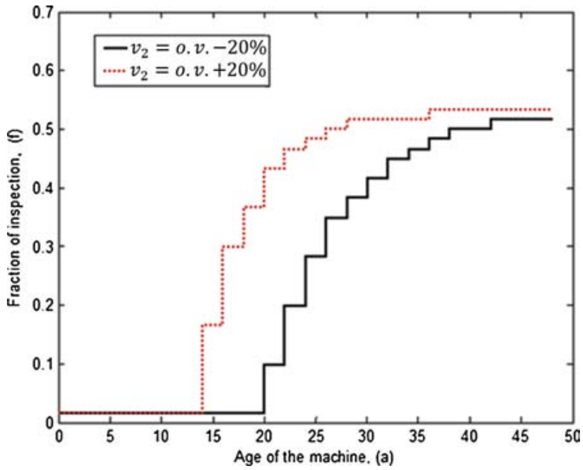


Fig. 24. Effect of the variation of the quality deterioration rate on the quality control policy.

- Policy-V: in this policy the inspection is not part of the optimization, the decision variables are only the production rate and the maintenance frequency which are adjusted according to the level of deterioration of the machine. Hence, we modified Policy-I to exclude inspection decisions from the optimization.

Regarding Policy-II, we have modified the model of Policy-I to allow the performance of 100 % inspection. Such inspection is conducted in all the units for any level of deterioration of the machine. With respect to Policy-III, the model determines the appropriate level of inspection to be conducted, in this case it resulted that $f_2^* = 23\%$ of the units are inspected. This fraction is obtained through optimization. In the case of Policy-IV, the stochastic process has been modified, since this policy leads to the case of a system with two modes $\Omega = \{1,2\}$, given that the optimization of major maintenance activities are not considered. Therefore, the set of feasibility control $\Gamma(\alpha)$, is modified to indicate that the minimization is conducted in terms of two decision variables of production and inspection. In Policy V we use the same deterioration dynamics as in Policy-I, with the exception that inspection decisions are disregarded.

7.1. Comparison of the control policies

In Table 4 we present the total incurred cost of the five policies considered in the comparative study, also we present the cost difference with respect to the proposed Policy-I with the aim to highlight the potential cost economies that could be obtained with our approach. Additionally, Table 4 also includes three quality indices, namely $\overline{FI}$, $\overline{AOQ}$ and AOQ_{max}. The indicator $\overline{FI}$ is calculated adding the fraction of inspection implemented in each age considered in the deterioration process presented in Fig. 7a, and then dividing the result by the age limit considered in the comparison. Further the models reports the average outgoing quality $\overline{AOQ}$, that is calculated in a similar fashion than the previous indicator. Table 4 also reports the maximum value of the indicator AOQ observed for each policy, denoted by AOQ_{max}. The critical age A_o that triggers major maintenance, also is reported in such table. The numerical approach presented in Appendix B was used to determine the results of Table 4.

The discussion of the results of the comparative study is as follows:

- Impact of Policy-II: under this strategy, 100 % inspection is always conducted regardless of the level of deterioration of the machine. Since all the units are inspected this has the result that the indicator $\overline{FI}$ increases significantly compared to the level of inspection implemented in Policy-I. Also, we note that Policy-II considerably reduces the defectives that reaches the final customer, where the quality indices decreases to $\overline{AOQ} = 0$ and $AOQ_{max} = 0$. Nevertheless, we note that Policy-II is the most expensive option of the study, since it reported a higher cost than Policy-I. The 100 % inspection strategy used in Policy-II leads to a higher total cost because of the performance of unnecessary inspection when the machine is in optimal conditions. Regarding the major maintenance policy, we note that since in Policy-II more inspection is conducted, then less major maintenance is needed, this increases the critical age A_o to 32 in order to delay the conduction of this activity. Summing up, Policy-II is 64.37 % more expensive than our approach.
- Impact of Policy-III: regarding the performance of this strategy, we note that the level of inspection conducted for any level of deterioration is $f_2^* = 23\%$. This strategy reduces the average fraction of units inspected $\overline{FI}$ compared to Policy-I, that inspects more units, especially when the level of deterioration is high. Therefore, we observe that with the conduction of less inspection, then more defectives reach the final customer, increasing indices $\overline{AOQ}$ and AOQ_{max} compared to Policy-I. Moreover, Policy-III is more expensive than Policy-I, because it performs more inspections than necessary at low levels of deterioration, and mainly because it does not perform enough inspection when the machine has high levels of deterioration, since in Policy-III the fraction of inspection is not dynamic. In Policy-III the sampling fraction always remains constant at a given value, and this is not effective in the context of deterioration, because more inspection is needed at higher levels of degradation. With respect to the major maintenance policy we note in Policy-III that less inspections are utilized, compared to the case of 100 % inspection of Policy-II, then this has the consequence that more major maintenance must be performed in order to mitigate the effects of deterioration, mainly observed in the presence of defectives. This reduces age A_o to 24. Policy-III leads to an increase of 23.10 % in the total cost.
- Impact of Policy-IV: the main characteristic of this strategy is that it disregards maintenance decisions from the optimization. We note from the results of Table 4 that the fact to only optimize production and quality control parameters leads to the conduction of more inspection, thus the indicator $\overline{FI}$ rises in Policy-IV. However, this measure increases the total cost, because the conduction of more inspection implies additional costs. Additionally, in Policy-IV at performing more inspection,

Table 4
Comparative study.

Description	Quality indices			Maintenance Indice A_o	Total Cost* ($)	Cost difference Δ-Cost (%)
	$\overline{FI}$(%)	$\overline{AOQ}$(%)	AOQ_{max} (%)			
Policy-I	**33.60**	**11.02**	**14.63**	**18**	**292.62**	–
Policy-II	100.00	0	0	32	480.97	+64.37 %
Policy-III	23.00	15.14	24.81	24	360.23	+23.10 %
Policy-IV	35.60	10.45	14.94	∞	364.53	+24.58 %
Policy-V	–	18.54	30.00	10	406.41	+38.89 %

less defectives reaches the final customer, reducing then the indices $\overline{AOQ}$. However, in Policy IV, the conduction of major maintenance is delayed to the end of the age limit, $A_o = \infty$ and this has negative consequences in the total cost, since it increases. Based on the results, we find that major maintenance decisions are closely related to production and inspection strategies and need to be addressed simultaneously. Thus, the joint optimization of these three key strategies has more economic benefits to the decision maker, as observed in the total cost of Policy-I. The dissociation of major maintenance decisions in Policy-IV leads to an increment in the total cost of 24.58 %.

- Impact of Policy-V: in this case inspection is not conducted, this policy only optimizes production and major maintenance rates. Unfortunately, Policy-V has the negative outcome that a considerable number of defectives reaches the final customer, because there is no mean to detect defectives without the conduction of inspection and prevent their presence only with the conduction of major maintenance is not economical feasible given the high cost of this maintenance option. Thus, we note

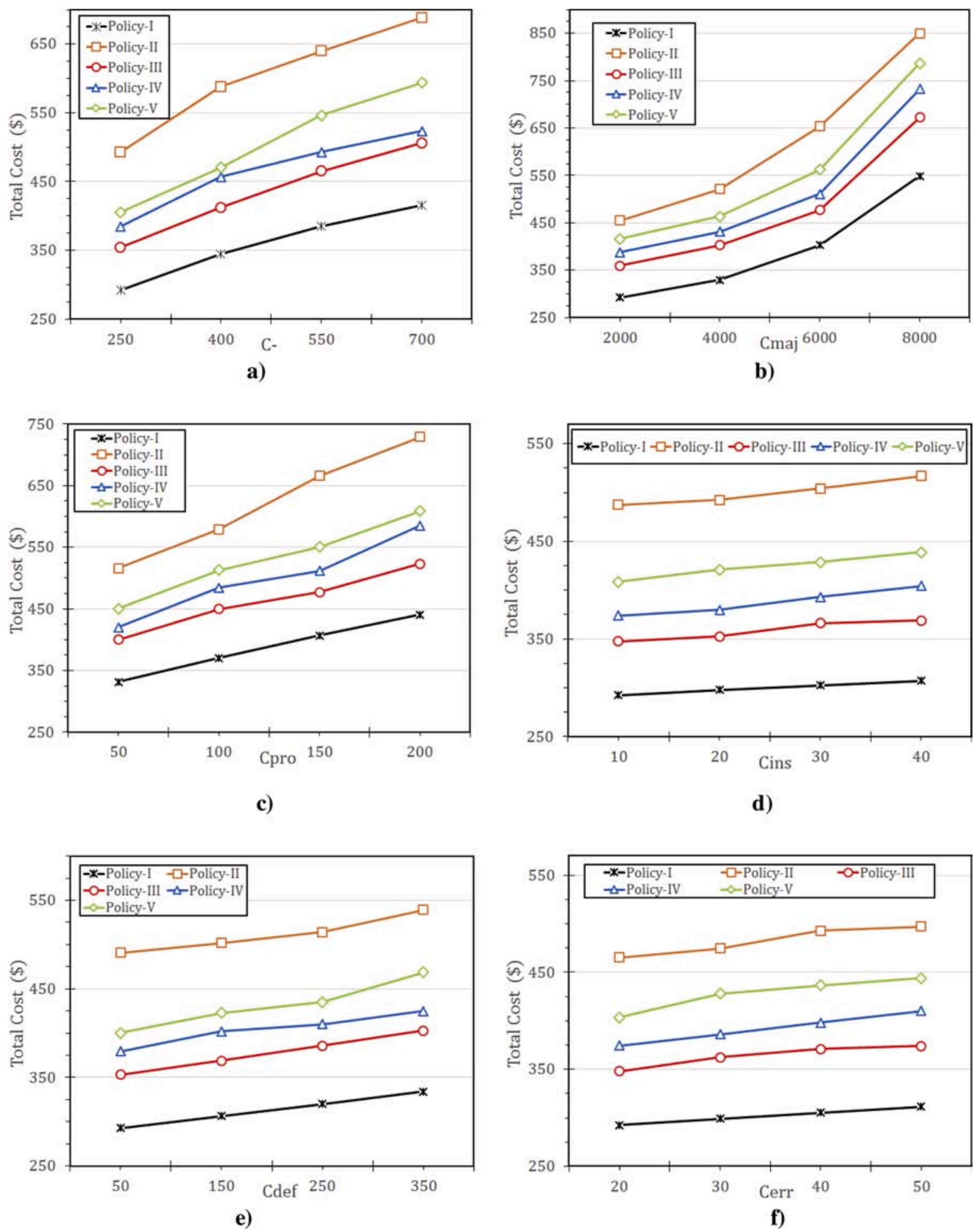


Fig. 25. Effect of the variation of several costs on the total cost for the considered policies.

that the quality indices $\overline{AOQ}$ and AOQ_{max} are the highest of the comparison. Additionally, we observe that in this context of less inspection conduction, more major maintenance must be performed to reduce the presence of defectives, therefore the critical age for major maintenance reduces to $A_o = 10$. From the results presented in Table 4, it is evident that the integration of production, quality and major maintenance provides more economic benefits that the case where the fraction of inspection is disregarded from the optimization. Policy-V resulted to be 38.89 % more expensive that Policy-I.

7.2. Comparison of the control policies for wide-range of cost and system parameters

In order to complement the comparative study, we perform more analysis of the performance of the considered five control policies in terms of the total incurred cost. In particular we analyze the impact of the variation of the set of costs parameters discussed in the previous section. Additionally we analyze the effect of the variation of two system parameters such as the intensity of the failure rate and the rate of generation of defectives. The analysis is presented in Figs. 25 and 26, where for each policy a solid line presents the total cost. The purpose of the comparison is to highlight that the proposed control policy provides significant cost economies than other common policies adapted from the literature.

From the observation of Fig. 25(a) we note that when the backlog cost increases the difference between the total incurred cost of the proposed control policy and the total cost of Policies II-IV increases. It is evident to see that the proposed Policy-I reported the lowest total cost of the comparison, mainly because it only inspects a fraction of the defective units, avoiding unnecessary inspection when the machine is in low levels of deterioration and also because it progressively adjust the production threshold and the level of inspection. Moreover, the results of Fig. 25(b) shows that the increment of the major maintenance cost, leads to the increment of the total cost in all the policies, because when C_{maj} increases, the conduction of major maintenance is delayed, and so the machine reaches higher levels of deterioration, generating then more defectives which increases the total cost. Fortunately, the proposed policy managed to provide a lower cost than the other policies at performing more inspection when the major maintenance is delayed and avoiding extra costs due to the presence of defectives.

Increasing the production cost as presented in Fig. 25(c), has the consequence of increasing the total cost. Because at increasing C_{pro} less inventory is permitted and so the machine operates less time at its maximum rate, thus it deteriorates less. In this context the conduction of major maintenance is delayed, and so the machine generates more defectives increasing the total cost. The proposed policy reported a lower total cost because it performs more inspection when major maintenance is delayed, and this avoids that more defectives reaches the final customer and their respective penalty. As can be seen from Fig. 25(d) when the inspection cost increases the total cost of the considered policies increases. Because inspections activities are more expensive. Nevertheless the proposed policy reported the lowest cost because despite that at increasing C_{ins} less inspection is conducted, Policy-I compensates with the conduction of more major maintenance, and this reduces considerably the presence of defectives and the reduction of further costs.

In Fig. 25(e) when there is a rise of the defectives cost, then the total cost increases for all the policies because there is more penalty when a defective unit reaches the final customer. However, Policy-I is less expensive because at increasing C_{def} it recommends two measures, it increases the rate of inspection and also it performs more major maintenance. With this less defectives are generated. Moreover in Fig. 25(f) if there is an increase in the cost of error of inspection, then the total cost of the policies increases. Since errors are more severely penalized. At increasing C_{err} it is logical that less inspection is conducted, but Policy-I reported a lower cost because it compensates such reduction with the conduction of more major maintenance.

Additionally, we complement the analysis with the variation of two system parameters related with the trend of the failure rate and the trend of the defectives rate, defined by the parameters η_2 and ν_2 as previously discussed, (see Eq. 9, Eq. 10, Figs. 3 and 4). In particular, in Fig. 26(a) we note that when the parameter η_2 increases, the total cost increases since the machine experiences more failures and more inventory is needed as protection. Policy-I achieves to obtain the best option for the total cost because at increasing η_2 it performs more major maintenance to rejuvenate the machine and restore the failure rate. Also more inspection is conducted. Regarding the increment of the quality deterioration rate presented in Fig. 26(b), it leads to increase the total cost because more defectives are generated. But the proposed policy obtains the lowest cost because as it performs two countermeasures, it performs more major maintenance to eliminate the generation of defectives and also it inspects more units.

As can be noted in the analysis of Figs. 25 and 26, the proposed policy reported a lower cost than the rest of policies, since Policy-I incorporates the strong interactions between the three key strategies of production-quality-maintenance, and mainly because it progressively adjust such policies based on the level of deterioration of the machine. This comparison confirms the usefulness of determining these three strategies simultaneously and that common policies that dissociates decisions leads to suboptimal solutions.

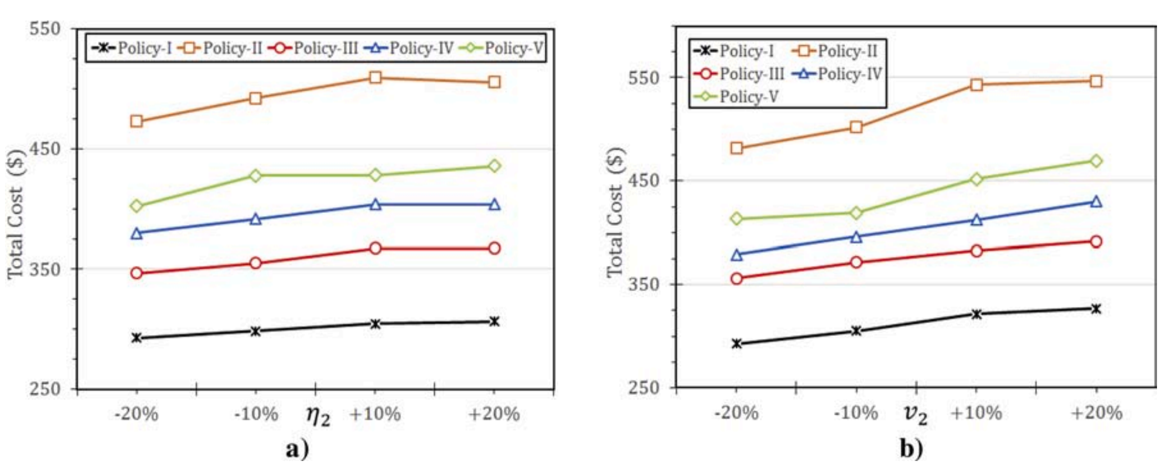


Fig. 26. Effect of the variation of two system parameters on the total cost for the considered policies.

8. Conclusion

This paper investigates the production planning, quality control and major maintenance control problem for an unreliable manufacturing system subject to quality and reliability deterioration. It is motivated by the need for better quality inspection strategies in cases where unreliable production units employ maintenance actions to restore the effects of the deterioration process. The problem was addressed using a stochastic optimal control approach considering two state variables denoted by the stock level and the age of the production unit. Optimality conditions were established in the form of HJB equations and a numerical method was used to approximate the continuous problem by a discrete counterpart. Our approach has allowed to develop a novel joint control policy that considers the set of effects of the deterioration process and the quality level constraint required by customers in the determination of the control parameters. A numerical example was conducted to illustrate the proposed approach, where we noted that our policy adjusts the production threshold and the inspection level in function of the degree of deterioration of the machine. An extensive sensitivity analysis has been conducted to test the robustness of the proposed joint control policy. In such analysis we observed that the obtained results are logical and consistent and they enable us to confirm the structure of the control policy, since we noted that it is well characterized by their control parameters in all the assessed scenarios. The results show a strong interaction between inspection and major maintenance activities and also, we noted that when the quality constraint were more severe, more inspection were conducted in order that less defectives reached the final customer. Additionally, the results presented in the comparative study section, show that the proposed control policy provided more benefits in terms of cost savings than traditional policies that employ 100 % inspection or policies that do not adjust the level of inspection according to the degree of deterioration of the machine or policies that dissociate maintenance decisions from the optimization. Also, we analyze the case where the fraction of inspection is not included in the optimization. However, our integrated model provides the best economical results in all the analyzed cases. A possible extension of this work can be the integration of imperfect preventive maintenance in the control strategy. Since such activity is commonly used in modern production systems and it is more realistic than perfect preventive maintenance strategies.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Optimality conditions

In this section the procedure to derive the HJB equations is detailed. Its relevance is that such expression defines the optimality conditions of the model, if we can solve it, then we can derive the optimal controls (u^*, f^*, ω^*) that yields to the minimum cost. The HJB Equations can be derived based on the *principle of optimality*, for instance if $V(\cdot, t)$ denotes a cost-to-go function at time t, then Eq. (20) takes the form of Eq. (A.1)

$$V(\alpha(t), x(t), a(t), t) = \min_{\substack{u(s),f(s),\omega(s)\\ t\le s\le\infty}} E\left\{ \int_t^\infty e^{-\rho(s-t)} G\left(\alpha(s), x(s), a(s), u(s), f(s), \omega(s), s\right) ds \,\middle|\, \alpha(t), x(t), a(t) \right\} \tag{A.1}$$

If the control is optimal in the time interval $[t, \infty]$ with initial conditions $\alpha(t),\ x(t),\ a(t)$ then based on the principle of dynamic programming it is also optimal in the time interval $[t, t+\delta t]$ at time $t+\delta t > t$ with initial conditions $\alpha(t+\delta t),\ x(t+\delta t),\ a(t+\delta t)$. Thus at breaking up the integral of Eq. (A.1) for any δt we have

$$V(\alpha(t), x(t), a(t), t) = \underset{\substack{u(s),f(s),\omega(s)\\ t\le s\le\infty}}{min} E\left\{ \begin{array}{c} \int_t^{t+\delta t} e^{-\rho(s-t)} G(\alpha(s), x(s), a(s), u(s), f(s), \omega(s), s) ds \\ + \\ \int_{t+\delta t}^{\infty} e^{-\rho(s-t)} G(\alpha(s), x(s), a(s), u(s), f(s), \omega(s), s) ds \end{array} \middle|\, \alpha(t), x(t), a(t) \right\} \tag{A.2}$$

Given that the integral in the interval $[t+\delta t, \infty]$ is the value function $V(\alpha(t+\delta t), x(t+\delta t), a(t+\delta t), t+\delta t)$. We obtain the one-step counterpart of $V(\alpha(t), x(t), a(t), t)$ in the interval $[t,\ t+\delta t]$:

$$V(\alpha(t), x(t), a(t), t) = \underset{\substack{u(s),f(s),\omega(s)\\ t\le s\le t+\delta t}}{min} E\left\{ \begin{array}{c} \int_t^{t+\delta t} e^{-\rho(s-t)} G(\alpha(s), x(s), a(s), u(s), f(s), \omega(s), s) ds \\ + \\ e^{-\rho\delta t} V(\alpha(t+\delta t), x(t+\delta t), a(t+\delta t), t+\delta t) \end{array} \middle|\, \alpha(t), x(t), a(t) \right\} \tag{A.3}$$

The cost function $G(\cdot)$ is continuous and treated as constant in the interval $\le s \le t+\delta t$. Additionally, the discount factor over δt is $e^{-\rho\delta t} = 1 - \rho\delta t + o(\delta t)$ and its integral in the time interval $t \le s \le t+\delta t$ is

$$\int_t^{t+\delta t} e^{-\rho(s-t)} ds = -\frac{1}{\rho}\left(e^{-\rho(t+\delta t-t)} - e^{-\rho(t-t)}\right) = \delta t + o\delta t \tag{A.4}$$

We can employ the conditional expectation operator $\widetilde{E}$, where for any function $H(\alpha)$, $\widetilde{E}\{H(\alpha(t+\delta t))\} = E\{H(\alpha(t+\delta t)|\alpha(t)\}$, then at assuming that the value function $V(\cdot)$ is differentiable, for small δt leads to:

$$\begin{aligned} & V(\alpha(t), x(t), a(t), t) = \\ & \min_{u(t), f(t), \omega(t)} \widetilde{E} \left\{ \begin{array}{c} G(\alpha(t), x(t), a(t), u(t), f(t), \omega(t))\delta t + (1-\rho\delta t)(V(\alpha(t+\delta t), x(t), a(t), t) \\ + \frac{\partial}{\partial x} V(\alpha(t+\delta t), x(t), a(t), t)\delta x\left(t\right) + \frac{\partial}{\partial a} V(\alpha(t+\delta t), x(t), a(t), t)\delta a\left(t\right) \\ + \frac{\partial}{\partial t} V(\alpha(t+\delta t), x(t), a(t), t)\delta t \Big) \end{array} \right\} \end{aligned} \tag{A.5}$$

We must expand the conditional expectation in Eq. (A.5) using the expansion $\widetilde{E}H(\alpha(t+\delta t)) = H(\alpha(t)) + \sum_j H(j)\lambda_{j\alpha(t)}\delta t + o(\delta t)$ where the term $o(\delta t)$ is negligible with respect to δt. After some transformation we get:

$$\begin{aligned} & \rho V(\alpha(t), x(t), a(t), t) - \tfrac{\partial}{\partial t} V(\alpha(t), x(t), a(t), t) = \\ & \min_{u(t), f(t), \omega(t)} \left\{ \begin{array}{c} G(\alpha(t), x(t), a(t), u(t), f(t), \omega(t)) + \frac{\partial}{\partial x} V(\alpha(t), x(t), a(t), t)\frac{dx(t)}{dt} \\ + \frac{\partial}{\partial a} V(\alpha(t), x(t), a(t), t)\frac{da(t)}{dt} \\ + \sum_j \lambda_{j\alpha(t)} V(j, x(t), a(t), t) \end{array} \right\} + o(\delta t) \end{aligned} \tag{A.6}$$

By considering that a steady-state distribution exists for α and that $V(\alpha, x, a, t) \rightarrow V(\alpha, x, a)$ as $t \rightarrow \infty$ and $\partial V/\partial t \rightarrow 0$, we finally obtain the HJB equations:

$$\begin{aligned} & \rho V(\alpha, x, a) = \\ & \min_{u(t), f(t), \omega(t)} \left\{ \begin{array}{c} G(\alpha, x, a, u, f, \omega) + \frac{\partial}{\partial x} V(\alpha, x, a)\frac{dx}{dt} \\ + \frac{\partial}{\partial a} V(\alpha, x, a)\frac{da}{dt} \\ + \sum_{j'} \lambda_{j\alpha} V(j, x, \varphi(\xi, a)) \end{array} \right\} \end{aligned} \tag{A.7}$$

Where $\varphi(\xi, a)$ denotes the reset function that defines the benefit of the repair and restores the age of the machine to as-good-as-new conditions after maintenance. Hence at a jump time σ for the process $\xi(t)$, we define this function as follows:

$$\varphi(\xi, a) = \begin{cases} 0 & \text{if } \xi(\sigma^+) = 1 \text{ and } \xi(\sigma^-) = 3 \\ a(\sigma^-) & \text{otherwise} \end{cases} \tag{A.8}$$

The optimal controls (u^*, f^*, ω^*) obtained from the HJB Eq. (A.7) are optimal since such equations are a necessary and sufficient condition for optimality, as noted in Gershwin [37] and Rivera-Gómez et al. [24]. This is the fundamental equation on which the Kushner's approach is based.

Appendix B. Numerical approach

In this appendix we detail the specialized numerical method, based on the Kushner's, approach, applied to find a solution to the HJB Eq. (21). The Kushner's, approach, who was proposed by Kushner and Dupuis [36], is a numerical method devoted to stochastic control problems in continuous time such as the model developed in this paper. The HJB Eq. (21) in most of the cases yields to intractable solutions due to the stochastic dynamics of the manufacturing system and the deterioration process defined by the age of the machine. We circumvent this complexity through the Kushner's approach, more precisely, such technique consists in using an approximation for the gradient of the value function $V(\cdot)$ based on finite differences. In the Kushner's technique, the value function $V(\cdot)$ is approximated by a discrete function $V_h(\cdot)$ and the partial derivatives of the first-order $\frac{\partial V}{\partial x}$ and $\frac{\partial V}{\partial a}$ are described by the following expressions:

$$\frac{\partial V}{\partial x}(\alpha, x, a) = \begin{cases} \frac{1}{h_x}(V_h(\alpha, x+h_x, a) - V_h(\alpha, x, a)) & \text{if } \dot{x} \geq 0 \\ \frac{1}{h_x}(V_h(\alpha, x, a) - V_h(\alpha, x-h_x, a)) & \text{otherwise} \end{cases} \tag{B.1}$$

and

$$\frac{\partial V}{\partial a}(\alpha, x, a) = \frac{1}{h_a}(V_h(\alpha, x, a+h_a) - V_h(\alpha, x, a)) \tag{B.2}$$

where h_x and h_a denote the length of the finite difference intervals of the state variables x and a. Then the above method was used to generate the following equation:

$$\frac{\partial V}{\partial x}(\cdot)(\dot{x}) = \begin{bmatrix} \frac{|\dot{x}|}{h_x}V_h(\alpha, x+h_x, a)\cdot Ind\{\dot{x}\geq 0\} \\ +\frac{|\dot{x}|}{h_x}V_h(\alpha, x-h_x, a)\cdot Ind\{\dot{x}<0\} \\ -\frac{|\dot{x}|}{h_x}V_h(\alpha, x, a) \end{bmatrix} \tag{B.3}$$

With

$$Ind\{P(\cdot)\} = \begin{cases} 1 & if\ P(\cdot)\ is\ true \\ 0 & otherwise \end{cases}$$

where $P(\cdot)$ is a given proposition. If we consider that the expression $Q(\cdot)V(\alpha, x,\ \varphi(\xi,a)(\alpha)$ leads to define

$$Q(\cdot)V(\alpha, x,\ \varphi(\xi,a)\left(\alpha\right) = \sum_{\alpha\neq\acute{\alpha}}\lambda_{\alpha\acute{\alpha}}(\cdot)V_h(\acute{\alpha}, x, \varphi(\xi,a)) - \sum_{\alpha=\alpha}\lambda_{\alpha\alpha}(\cdot)V_h(\alpha, x, \varphi(\xi,a)) \tag{B.4}$$

With the approximation (B.3) and expression (B.4) we can rewrite the HJB Eq. (21) in terms of $V_h(\cdot)$ as described in the next expression:

$$\rho V_h(\alpha,x,a) = \min_{\substack{(u,f,\omega)\in\Gamma(\alpha)\\ AOQ(a)\leq AOQL}} \begin{bmatrix} G(\cdot)+\frac{|\Theta_x|}{h_x}V_h(\alpha,x+h_x,a)\cdot Ind\{\dot{x}\geq 0\} \\ \frac{|\Theta_x|}{h_x}V_h(\alpha,x-h_x,a)\cdot Ind\{\dot{x}<0\} - \frac{|\Theta_x|}{h_x}V_h(\alpha,x,a)+\frac{\Theta_a}{h_a}V_h(\alpha,x,a+h_a) \\ +\sum_{\alpha\neq\acute{\alpha}}\lambda_{\alpha\acute{\alpha}}(\cdot)V_h(\acute{\alpha},x,\varphi(\xi,a)) - \sum_{\alpha=\alpha}\lambda_{\alpha\alpha}(\cdot)V_h(\alpha,x,\varphi(\xi,a)) \end{bmatrix} \tag{B.5}$$

After some manipulations Eq. (B.5) becomes

$$V_h(\alpha,x,a) = \min_{\substack{(u,f,\omega)\in\Gamma(\alpha)\\ AOQ(a)\leq AOQL}} \left[\left(\rho+\frac{|\Theta_x|}{h_x}+\frac{\Theta_a}{h_a}+|\lambda_{\alpha\alpha}|\right)^{-1} \left(\begin{matrix} G(\cdot)+\frac{\Theta_a}{h_a}V_h(\alpha,x,a+h_a)+\ \frac{|\Theta_x|}{h_x}V_h(\alpha,x+h_x,a)\cdot Ind\{\dot{x}\geq 0\} \\ +\frac{|\Theta_x|}{h_x}V_h(\alpha,x-h_x,a)\cdot Ind\{\dot{x}<0\} \\ +\sum_{\alpha\neq\acute{\alpha}}\lambda_{\alpha\acute{\alpha}}(\cdot)V_h(\alpha,x,\varphi(\xi,a)) \end{matrix}\right)\right] \tag{B.6}$$

Where $\Theta_x = [1 - f(\cdot)\cdot\beta(a)]\cdot u_T - \frac{d}{1-AOQ(a)}$ and $\Theta_a = k_1\cdot u(t)$. Eq. (B.6) defines a discrete Markovian decision control problem with finite state space and with finite action space. The solution of $V_h(\alpha,x,a)$ is an approximation which will converge to the solution $V(\alpha,x,a)$ of Eq. (20) as $h_x\rightarrow 0$ and $h_a\rightarrow 0$. In this paper the policy iteration technique is used to obtain a solution of the approximating optimization problem. The algorithm of this technique can be consulted in Kushner and Dupuis [36] and references therein.

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