MULTI-STAGE STOCHASTIC MODELS FOR PRODUCTION PLANNING OF A FURNITURE MANUFACTURING COMPANY UNDER UNCERTAINTY

José Emmanuel Gómez-Rocha[1], Héctor Rivera-Gómez[1]*, Eva Selene Hernández-Gress[2], Antonio Oswaldo Ortega-Reyes

[1]UNIVERSIDAD AUTÓNOMA DEL ESTADO DE HIDALGO, Área Académica de Ingeniería, Profesor-Investigador, Carretera Pachuca-Tulancingo km 4.5, Ciudad del Conocimiento, Mineral de la Reforma, Hidalgo, México, 42184. Telf: 7172000, extensión: 4001.

[2]INSTITUTO TECNOLÓGICO Y DE ESTUDIOS SUPERIORES DE MONTERREY, Campus Hidalgo, Escuela de Ingeniería y Ciencias, Profesor-Investigador, Boulevard Felipe Ángeles no. 2003, Pachuca de Soto, Hidalgo, México, 42080. *hriver06@hotmail.com

Received: 28/Feb/2020–Reviewing: 3/Mar/2020--Accepted: 28/Apr/2020 - DOI: http://dx.doi.org/10.6036/MN9702

TO CITE THIS ARTICLE:

GOMEZ-RONCHA, Jose Emmanuel, RIVERA-GOMEZ, Hector, HERNANDEZ-GRESS, Eva Selene et al. MULTI-STAGE STOCHASTIC MODELS FOR PRODUCTION PLANNING OF A FURNITURE MANUFACTURING COMPANY UNDER UNCERTAINTY. DYNA Management, January-December 2020, vol. 8, no. 1, [13 p.]. DOI: http://dx.doi.org/10.6036/MN9702

ABSTRACT:

In this article, two multi-stage stochastic linear programming models are developed where the uncertainty of the random variable is modeled using a continuous probability distribution or a discrete probability distribution. The developed models are applied to an aggregate production plan for a furniture manufacturing company located in the state of Hidalgo, Mexico, which has important customers such as chain stores with presence throughout the country. Production capacity is defined as the random variable of the model. Uncertainty is modeled through a scenario tree in a multi-stage environment. The main purpose of this research is to determine a feasible solution to the aggregate production plan in a reasonable computational time. The Lingo software was used to find a solution of the model, using the Branch and Bound solver (B-and-B). Furthermore the two developed models were compared in terms of accuracy and computational time. The study is complemented with an extensive sensitivity analysis, where it is assessed the effect of several costs on the optimal solution. Besides, the impact of the service level constraint on the decision variables is analyzed.

***Keywords**: Aggregate production planning, multi-stage stochastic optimization, scenario tree.*

1.- INTRODUCTION

The aggregate production plans have a very important role in manufacturing industries, within the global planning of an organization it is conducted in the medium-term, being common periods from 3 to 18 months. Production plans seek to determine the optimum levels of production, hiring, dismissal, inventories, backlog, subcontracting, etc. In this article an aggregate production plan is developed for a furniture company located in the state of Hidalgo, Mexico. In particular, two models based on the stochastic mixed-integer linear programming were developed.

There are several approaches used in the literature to develop aggregate production plans under uncertainty and these can be classified into six broad categories: stochastic mathematical programming, possibilistic programming, diffuse mathematical programming, simulation models, metaheuristics and evidential reasoning. Some authors have focused on multi-objective stochastic optimization. For example, Nowak [1] proposed a model that combines linear mathematical programming with multi-objective optimization, simulation and an interactive approach with uncertain demand. Jamalnia et al. [2] presented a non-linear stochastic optimization model with multiple objectives for an aggregate production plan under uncertainty. Zhao et al. [3] presented a case study for a pharmaceutical industry where they optimized the amount of production, minimizing the duration of clinical trials and operating costs. Rakes, Franz, and Wynne [4], and Chen and Liao [5] also developed models considering the uncertainty in production plans with multi-objectives. Other researchers have focused on mixed linear and integer linear stochastic optimization models where the system is subject to uncertainty. Birge and Louveaux [6] and Wagner & Whitin [7], are considered the first researchers who studied production plans under uncertainty. Zanjani et al. [8] presented a multi-stochastic mixed linear mathematical programming model for the production of sawmills in the timber industry in Canada. Huang [9] presented a model for production planning based in the multi-stage stochastic mathematical programming.

Nonlinear stochastic optimization has been studied by several authors such as Mirzapour Al-e-hashem et al. [10] who proposed a model to study the decision problem of an aggregate production plan in the presence of uncertainty. Nasiri et al. [11] presented a model for a production and distribution plan in a three-stage supply chain considering suppliers, production centers and customers. Similarly, Nasiri et al. [12] proposed a stochastic model for a supply chain and later extend their model to a multi-stage system in Nasiri et al. [13]. In the work of Ning et al. [14] a multiproduct non-linear application model was presented where market demand and production cost are uncertain. Lieckens and Vandaele [15] considered the demand and the delivery time as random variables in their model. Robust optimization has been used to model uncertainty in aggregate plans such as in Leung and Wu [16], Kanyalkar and Adil [17], Mirzapour Al-e-hashem, Malekly and Aryanezhad [18], Mirzapour Al-e -hashem, Aryanezhad and Sadjadi [19], Makui, Heydari, Aazami and Dehghani [20]. Where robust optimization techniques were used to deal with aggregate production plans under uncertainty. The scenario tree is a feasible way to discretize the underlying stochastic data over time in a stochastic optimization problem. This tree allows to determine feasible solutions in reasonable times. Hu and Hu [21], used a scenario tree to optimize a sequencing problem and lot sizing under uncertain demand. Körpeoglu, Yaman and Aktürk [22] also used a scenario tree to solve a production problem.

The main objective of this paper is to compare two multi-stage stochastic optimization models for the production planning of a furniture company that is subject to random production capacity. We examine two different methods to model such uncertainty. We employ a continuous probability distribution in the first stochastic model. The second model uses a discrete probability function to model randomness. A comparative study is performed in terms of resolution time and the number of iterations to determine the best option that provides accurate results in an acceptable period of time. The paper is complemented with an extensive sensibility analysis and the assessment of the impact of the service level constraint.

The rest of the paper consists of the following sections. Section 2 introduces the formulation of the deterministic and stochastic models under study. Material and methods are presented in Section 3. A numerical instance is developed in Section 4. In Section 5 an extensive sensitivity analysis is conducted. Finally Section 6 concludes the paper.

2. - DESCRIPTION OF THE MODEL

This paper presents two stochastic linear programming models applied to an aggregate production plan of a furniture company. The deterministic model of base of the study defines the planning horizon in T months and the objective function seeks to reduce the total cost comprising the raw material, the labor, the inventory level, and the backlog costs per month. It is also considered a service level of 90% per month as a management policy.

2.1.- FORMULATION OF THE DETERMINISTIC MODEL

The deterministic model considers the following assumptions:

- The demand is known for all periods.
- Backlog may exist in the company, but it is penalized.
- Backlogged units must be satisfied the next period.
- There is enough raw material for production.

The notation used for the deterministic model is shown in Table 1.

Indices:	
$\boldsymbol{T}$	Time horizon of the aggregate plan.
Parameters:	
$\boldsymbol{d_t}$	Monthly product demand for period $\boldsymbol{t}$
$\boldsymbol{C_p}$	Salary of the production workers per month
$\boldsymbol{C_F}$	Firing cost
$\boldsymbol{C_R}$	Hiring cost
$\boldsymbol{C_I}$	Inventory cost
$\boldsymbol{C_S}$	Backlog cost
$\boldsymbol{C_X}$	Raw material and indirect cost
$\boldsymbol{k}$	Monthly production capacity per worker.
Decision variables:	
$\boldsymbol{w_t}$	Number of workers per month
$\boldsymbol{p_t}$	Number of workers assigned to production per month
$\boldsymbol{r_t}$	Number of hired workers per month
$\boldsymbol{f_t}$	Number of fired workers per month
$\boldsymbol{x_t}$	Number of parts to be produced per month
$\boldsymbol{I_t}$	Number of parts in the inventory per month
$\boldsymbol{I_\alpha}$	Safety inventory per month
$\boldsymbol{s_t}$	Backlog per month

Table 1. Notation of the deterministic model

The objective function of the deterministic model minimizes the total cost, considering the labor cost, the layoffs cost, the hiring cost, the inventory costs, the backlog cost and the raw material cost, as defined in the following equation:

$$min\ Z = \sum_{t=1}^{T} p_t C_P + \sum_{t=1}^{T} f_t C_F + \sum_{t=1}^{T} r_t C_R + \sum_{t=1}^{T} I_t C_I + \sum_{t=1}^{T} x_t C_X + \sum_{t=1}^{T} s_t C_S \tag{1}$$

The model constraints are the following:

$$w_t = w_{t-1} + r_{t-1} - f_{t-1}\ \forall\ t = 1, \dots, T \tag{2}$$

$$w_t = p_t + f_t\ \ \forall\ t = 1, \dots, T \tag{3}$$

$$x_t + I_{t-1} = d_t + s_{t-1} + I_t - s_t\ \forall\ t = 1, \dots, T \tag{4}$$

$$x_t \le kp_t\ \ \forall\ t = 1, \dots, T \tag{5}$$

$$I_t \ge I_\alpha\ \forall\ t = 1, \dots, T \tag{6}$$

$$d_t - s_t \ge .90 d_t\ \forall\ t = 1, \dots, T \tag{7}$$

$$w_t,\ p_t,\ r_t,\ f_t,\ x_t,\ I_t,\ s_t,\ d_t \ge 0\ \forall\ t = 1, \dots, T, t \in T \tag{8}$$

$$w_t,\ p_t, r_t,\ f_t,\ x_t,\ I_t,\ s_t,\ d_t \in \mathbb{Z}\ \forall\ t = 1, \dots, T \tag{9}$$

Constraint (2) focuses on the size of the workforce in the company, and it indicates that the total number of workers in period t, must be equal to those existing in period $t-1$, plus those hired in period t-1, minus those laid off in period $t-1$. Constraint (3) is about the allocation of the workforce, it simply defines how many workers will be assigned to production and the amount of workers that will be laid off in period t. Constraint (3) complements the balance over the number of workers denoted in Equation (2) and specifies the assignment of workforce in production and the number of workers to be fired each month. Constraint (4) refers to the balance of demand and inventory in the company, where what is produced in period t plus the inventory of period $t-1$, must be equal to the demand of period t, plus the backlog of period $t-1$ plus the inventory in period t minus the backlog of period t. Constraint (5) addresses the production capacity, it ensures that the workers assigned to production can manufacture the units required in period t. Constraint (6) defines that the inventory of the period is greater than the safety inventory defined by the company. Constraint (7) indicates that the service level is greater than or equal to 90% per month, this restriction is a company's policy. The expression (8), defines the constraint of non-negativity. The expression (9) indicates that the decision variables must be integers, which implies an integer stochastic programming model.

2.2.- FORMULATION OF THE STOCHASTIC MODEL

The previous deterministic model (1)-(9) is the base for developing a multi-stage stochastic model where the production capacity is the random variable. Two stochastic models were developed as follows:

- Model-I: in this model the random variable related with the production capacity is modeled with a continuous probability distribution. In this case the normal distribution was used.
- Model-II: in this model, the production capacity is associated with a discrete probability distribution with three possible values (low, medium and high), each value with a given probability.

The stochastic models have the following assumptions:

- For Model-I, the production capacity follows a normal distribution with mean 12 and standard deviation 2. These values were obtained through historical data of the company. The Arena ™ Input Analyzer was used to determine these values.
- For Model-II that uses the discrete distribution to model the random variable, the Gaussian quadrature method, Zanjani et al. [8], was used to determine the probabilities for the values low, medium and high, respectively.

The notation of the stochastic models is shown in Table 2.

Índices:	
$\boldsymbol{T}$	Time horizon of the aggregate plan.
N	Number of scenarios considered in the model
$\boldsymbol{\omega}$	Production capacity scenario
Parameters:	
$\boldsymbol{d_t}$	Monthly demand in period $\boldsymbol{t}$
$\boldsymbol{C_p}$	Salary of the production workers per month
$\boldsymbol{C_F}$	Firing cost
$\boldsymbol{C_R}$	Hiring cost
$\boldsymbol{C_I}$	Inventory cost
$\boldsymbol{C_S}$	Backlog cost
$\boldsymbol{C_X}$	Raw material and indirect cost
$\boldsymbol{k_t^\omega}$	Monthly production capacity per worker in scenario $\boldsymbol{\omega}$ in period $\boldsymbol{t}$ (random variable)
Variables de decisión:	
$\boldsymbol{w_t}$	Number of workers per month
$\boldsymbol{p_t}$	Number of workers assigned to production per month
$\boldsymbol{r_t}$	Number of hired workers per month
$\boldsymbol{f_t}$	Number of fired workers per month
$\boldsymbol{x_t}$	Number of parts to be manufactured per month

$\boldsymbol{I_t}$	Number of parts in the inventory per month
$\boldsymbol{I_\alpha}$	Safety inventory per month
$\boldsymbol{s_t}$	Backlog per month
$\boldsymbol{P^\omega}$	Probability of scenario $\boldsymbol{\omega}$ in the scenario tree, such probabilities satisfies the following equation

$$\sum_{\omega=1}^{N} P^{\omega} = 1 \qquad (10)$$

Equation (10) indicates that the sum of all the probabilities in the scenario tree must be equal to one.

Table 2. Notation of the stochastic model

The solution strategy used in the stochastic models is based on the generation of a scenario tree for the random variable. In each period the random variable can take a set of defined values with some probability and at the end of the planning horizon generates a set of s_i scenarios. For example, in Figure 1 if the random variable has three different values, thus nine scenarios are generated in two periods, each with a certain probability. The sum of all the probabilities of the nine scenarios must be equal to one.

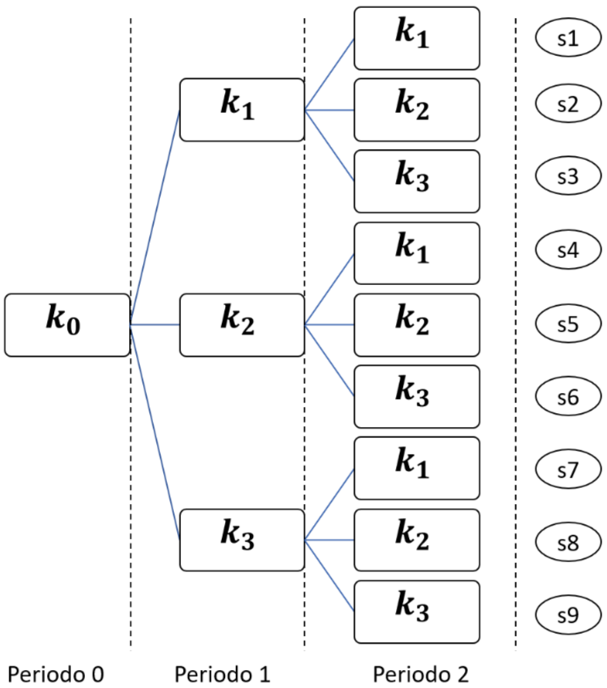

Fig. 1. Decision tree for the stochastic model

The objective function of the stochastic models optimize the costs for each scenario as follows:

$$\min Z = \sum_{\omega=1}^{N} P^{\omega} \sum_{t=1}^{T} p_t(\omega)\, C_P + \sum_{\omega=1}^{N} P^{\omega} \sum_{t=1}^{T} f_t(\omega)\, C_F + \sum_{\omega=1}^{N} P^{\omega} \sum_{t=1}^{T} r_t(\omega)\, C_R + \sum_{\omega=1}^{N} P^{\omega} \sum_{t=1}^{T} I_t(\omega)\, C_I + \sum_{\omega=1}^{N} P^{\omega} \sum_{t=1}^{T} x_t(\omega)\, C_X + \sum_{\omega=1}^{N} P^{\omega} \sum_{t=1}^{T} s_t(\omega)\, C_S \quad (11)$$

The constraints of the stochastic model are:

$$w_t(\omega) = w_{t-1}(\omega) + r_{t-1}(\omega) - f_{t-1}(\omega) \quad \forall\, t = 1, \dots, T, \omega = 1, \dots, N, \omega \in \Omega, t \in T \tag{12}$$

$$w_t(\omega) = p_t(\omega) + f_t(\omega)\ \forall\, t = 1, \dots, T, \omega = 1, \dots, N, \omega \in \Omega, t \in T \tag{13}$$

$$x_t(\omega) + I_{t-1}(\omega) = d_t(\omega) + s_{t-1}(\omega) + I_t(\omega) - s_t(\omega) \quad \forall\, t = 1, \dots, T, \omega = 1, \dots, N, \omega \in \Omega, t \in T \tag{14}$$

$$x_t(\omega) \le \boldsymbol{k_t^{\omega}}\, p_t(\omega)\ \forall\, t = 1, \dots, T, \omega = 1, \dots, N, \omega \in \Omega, t \in T \tag{15}$$

$$I_t(\omega) \ge I_\alpha\ \forall\, t = 1, \dots, T, \omega = 1, \dots, N, \omega \in \Omega, t \in T \tag{16}$$

$$d_t(\omega) - s_t(\omega) \ge .90 d_t(\omega) \forall\, t = 1, \dots, T, \omega = 1, \dots, N, \omega \in \Omega, t \in T \tag{17}$$

$$w_t(\omega),\ p_t(\omega),\ r_t(\omega), f_t(\omega), x_t(\omega), I_t(\omega), s_t(\omega), d_t(\omega) \ge 0 \quad \forall\, t = 1, \dots, T, \omega = 1, \dots, N\ , \omega \in \Omega, t \in T \tag{18}$$

$$p_t(\omega), r_t(\omega), f_t(\omega), s_t(\omega), I_t(\omega) \in \mathbb{Z}\ \forall\, t = 1, \dots, T \tag{19}$$

Constraint (12) deals with the size of the workforce in the company, where workers in period t must be equal to those that existed in period $t-1$ plus those hired in period $t-1$ minus those laid off in period t-1 for each scenario. Constraint (13) indicates how many workers will be assigned to production and those who will be laid off per period t in each scenario. Constraint (14) is about the balance of demand and inventory in the company, it states what must be produced in period t plus the inventory of period $t-1$, must be equal to the demand of period t plus the backlog of period $t-1$, plus the inventory of period t, minus the backlog of period t. Constraint (15) incorporates the random variable $\boldsymbol{k_t^{\omega}}$ into the model, and deals with the production capacity of the company and ensures that the workers assigned to production in period t are capable of producing the demand requirements. Constraint (16) indicates that the inventory in period t is greater than the safety inventory per month required by the management, in each scenario. Constraint (17) ensures that the service level in period t is greater than or equal to 90%, in all periods and all scenarios. Expression (18) is the restriction of non-negativity for the decision variables. Expression (19) indicates that the decision variables must be integers.

3.- MATERIAL & METHODS

The methodology used in this research consists of the following steps:

1. Data obtaining: historical data of the company was used to determine production parameters and associated costs. Furthermore, an ABC classification of products was conducted, where the best seller article was determined. In this case the best seller unit of the company is a rustic chair.
2. Deterministic mathematical modeling: a deterministic optimization model was developed. The company's policies of maintaining a minimum safety inventory each month and the policy of having a service level of at least 90% per month were taken into account in the formulation.
3. Determination of the demand and production capacity: A forecast of the product demand for six months was carried out in order to have demand data for the optimization. Several qualitative methods such as moving averages, double exponential smoothing and linear regression were used in this process. Furthermore these forecasts were evaluated to ensure a good fit of the data. Subsequently, with the historical production data, the average production capacity per worker was determined.
4. Generation of the first stochastic model: due to randomness, it was noted that the production capacity of the workers was not constant over time. Thus, based on historical production data, the Arena™ software Input Analyzer tool was used, observing that the production capacity follows a normal distribution with mean 12 and a standard deviation of approximately 2 as shown in Figure 2. The stochastic Model-I uses the normal distribution to model the random variable. The chi-square test and the

Kolmogorov-Smirnov test were used to assess the goodness fit of the proposed distribution and they serve us to validate the model.

```
        Distribution Summary

Distribution:   Normal
Expression:     NORM(12, 1.99)
Square Error:   0.014078

Chi Square Test
  Number of intervals   = 3
  Degrees of freedom    = 0
  Test Statistic        = 1.63
  Corresponding p-value < 0.005

Kolmogorov-Smirnov Test
  Test Statistic        = 0.128
  Corresponding p-value > 0.15
```

Fig. 2. Parameters of the continuous distribution

5. Generation of the second stochastic model: the Gaussian quadrature method was used to approximate the normal distribution of the previous step with a discrete probability distribution, as in Zanjani et al. [8]. This discrete distribution used in Model-II has three values (low, medium and high). The probabilities associated for such values were 0.16, 0.67 and 0.17, respectively, with $\mu = 12$ and $\sigma = 2$, and considering a range of $\mu \pm 1\sigma$.
6. Resolution and comparative study: An extensive sensitivity analysis was performed. Moreover, the impact of the service level constraint on the decision variables was analyzed.

4.- RESULTS

A Dell Inspiron 5570 computer with an Intel® Core ™ i5-8250U processor with 1.6GHz and 4Gb RAM with Windows 10 operating system was used. The Lingo 17.0 software was applied with 100 heuristics in the pre-solver of the integer optimization and the Branch and Bound method was used for the solution. Table 3 shows how the problem size grows in the number of variables when more periods are considered in the aggregate production plan. This growth is observed because the model has to generate first the equivalent deterministic model to determine a solution, and more variables are needed when the number of periods increases.

	Deterministic Model			Stochastic model					Equivalent deterministic		
period	**var.**	**int. var.**	**constr.**	**num. of sce.**	**rand. var.**	**var.**	**int. var.**	**constr.**	**var.**	**int. var.**	**constr.**
2	14	10	12	9	2	14	10	12	126	90	198
3	21	15	18	27	3	21	15	18	567	405	936
4	28	20	24	81	4	28	20	24	2268	1620	3852
5	35	25	30	243	5	35	25	30	8505	6075	14706
6	42	30	36	729	6	42	30	36	30618	21870	53586

Table 3. Size of the optimization models in function of the number of periods

As can be seen in Table 4, with the increase in the number of periods, it is more difficult to determine a solution when it is assumed that the random variable is associated with a continuous probability distribution such as the normal distribution. Therefore, alternative approaches are needed to determine a solution in a shorter period of time. This observation justifies the development of Model-II. In Table 4, we note that the number of iterations needed to determine a solution remains significantly larger in Model-I compared to the iterations needed in Model-II. Therefore more time is required to solve the models when the number of periods increases. This table also compares both stochastic models in terms of the CPU time that is the time needed for the computer to find a solution.

	CPU time (sec.)			Iterations			Type of solution		
Per.	**Determi-nistic Model**	**Model-I Stochas.**	**Model-II Stochas.**	**Determi-nistic Model**	**Model-I Stochas.**	**Model-II Stochas.**	**Determi-nistic. Model**	**Model-I Stochas.**	**Model-II Stochas.**
2	0.12	0.13	0.12	56	1722	1411	Global optimum	Global optimum	Global optimum
3	0.13	0.35	0.28	85	3975	3582	Global optimum	Global optimum	Global optimum
4	0.13	6.11	1.51	223	88319	17024	Global optimum	Global optimum	Global optimum
5	0.13	31.64	5.44	232	1373124	74061	Global optimum	Global optimum	Global optimum
6	0.13	6000*	68.3	339	57187059	446690	Global optimum	Feasible solution	Global optimum

Table 4. Comparison of the optimization models, "" denotes that the model was stopped at the end of 6000 sec.*

In Figure 3 we note that Model-I needs significantly more time to determine a solution than Model-II. The use of a continuous probability distribution in Model-I is more time demanding and for a six period model it was stopped after 6000 sec. because it did not find a global optimum solution.

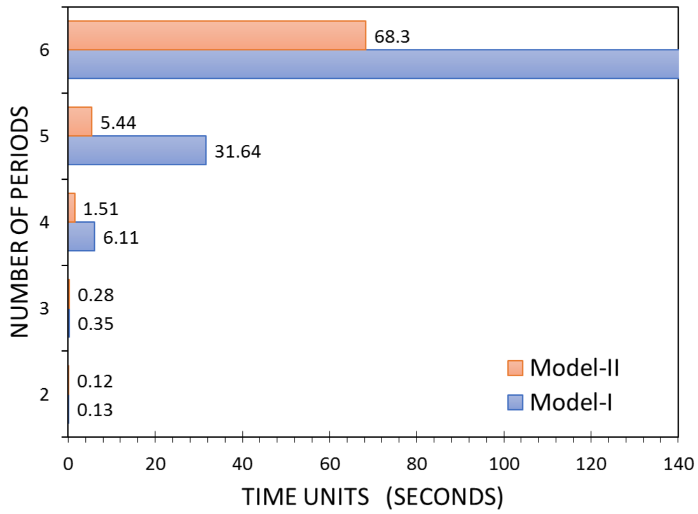

Fig. 3. CPU time

Table 5 shows the values of the stochastic optimization indicators EV, WS, and EVPI. The indicator EV denotes the expected value of the objective over all the scenarios of the model. The indicator WS reports the expected value of the objective if we could wait and see the outcomes of the random variable before making a decision. Obviously it is not possible in practice because we cannot anticipate uncertainty. The indicator EVPI is the absolute value of the difference EV-WS, and it measures the maximum amount a decision maker would be ready to pay in return of complete information about the future, it represents the loss of profit due to the presence of uncertainty. Therefore as we can noted in Table 5 that as the indicator EVPI is large in all cases then a stochastic model is justified. Also Table 5 presents the percentage difference between Model-I and Model-II in terms of the EV indice. We denote such difference as the gap, and from the obtained results we note that such gap is very low in all cases, and so we can state that the proposed approach of using a discrete probability distribution in Model-II is an accurate approximation of the continuous probability distribution of Model-I.

	EV		**WS**		**EVPI**		**Gap of EV (%)**
Periods	Model-I	Model-II	Model-I	Model-II	Model-I	Model-II	
2	538496	538636	465031.7	470732.3	73464.33	67903.67	0.0260%
3	831329	837645	731235.3	743614.3	100093.7	94030.74	0.7540%
4	1122591	1143415	996391	1016133	126200	127281.5	1.8212%
5	1449983	1478359	1282624	1317534	167358.7	160854.6	1.9194%
6	1765060	1787120	1559876	1599086	205184.2	188034.3	1.2344%

Table 5. EV, WS and EVPI indicators

In Figure 4 we note that the EVPI indicator is larger for Model-I in all the analyzed number of periods, indicating that a smaller cost can be achieved when a continuous distribution is used. However, the approximation provided by Model-II is acceptable since the gap difference is negligible.

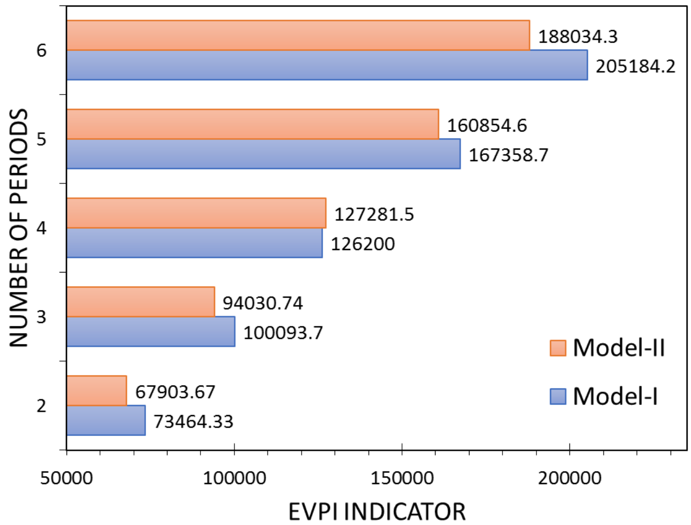

Fig. 4. EVPI indicator

5. - SENSITIVITY ANALYSIS

In this section, an extensive sensitivity analysis was carried out for the stochastic model with the aim to observe how sensitive the objective function and the decision variables are in terms of the variation of the set of costs. Model-II was used for conducting this analysis. Table 6 reports the value of the decision variables and the variation of the expected value of the objective function in the most likely scenario. The indices of the decision variables reported in Table 6 indicate their period.

Number	Parameter	Variation (%)	Decision variables						Δ-Cost (%)
			P_1	R_1	F_6	X_1	I_1	S_6	
Base case	-	-	32	1	0	320	170	33	-
1	C_p	50%	32	1	0	320	170	33	38.5816%
2		-50%	33	0	0	330	180	6	-37.847%
3	C_F	100%	32	1	0	320	170	33	0.0000%
4		-100%	34	0	3	340	190	33	-0.3277%
5	C_R	100%	33	0	0	330	180	6	1.6017%
6		-100%	30	4	0	300	150	6	-38.151%
7	C_I	200%	32	0	0	319	169	6	1.3687%
8		-100%	33	0	0	330	180	6	1.2685%
9	C_S	100%	33	0	0	330	180	6	1.3656%
10		-100%	33	0	0	329	179	6	1.2767%
11	C_X	100%	33	1	0	330	180	6	23.6708%
12		-100%	33	0	0	330	180	3	-21.038%
13	D_t	35%	45	0	0	450	178	9	38.4762%
14		-35%	21	0	0	210	183	4	-36.206%

Table 6. Sensitivity analysis

In Table 6, the sensitivity analysis was performed by varying the set of costs a certain percentage. The first analyzed parameter C_p is the cost of the salary of production workers; it can be seen in case 2 that if this cost is reduced by half, more workers are assigned to production and with this measure, more units are produced and so more inventory is accumulated. This considerably reduces backlog. The second parameter C_F refers to the cost of dismissal, we observe that if we reduce this cost, more layoffs are permitted in the company. However, more workers are assigned to production, thus more units are produced and more pieces are accumulated in the inventory. The third parameter C_R is the cost of hiring, in case 5 we note that when we increase this cost, no workers are hired in the company. Nevertheless, more workers are assigned to production and with this measure, more units are produced and more pieces are stocked in the inventory, reducing considerably backlog. The fourth analyzed parameter C_I, is the inventory cost. In case 8 we observe that if this cost is reduced, more workers are assigned to production, hence the company produces more pieces increasing the size of the inventory. The fifth parameter C_S is the backlog cost; from the obtained results it is observed in case 8, that when this cost increases, the company produces more units and this increases the size of the inventory in order to avoid the penalization of not being able to satisfy product demand on time. The sixth parameter C_X refers to the raw material cost, in case 11 it is evident that if this cost increases, more workers are assigned to production and so more employees are hired to reduce backlog and increase the profits of the company. The parameter D_t is the demand of the period, it is observed in case 13 that if the demand increases, more workers are assigned to production and with this countermeasure, the company produces more units and this considerably increases the level of inventory. It is worth mentioning that this sensitivity analysis was performed with the data provided by the company and it is evident that the analyzed parameters have a strong impact on the decision variables and the total cost.

5.1.- EFFECT OF THE SERVICE LEVEL CONSTRAINT

The service level constrain (17) has an important role in the stochastic model, which is to ensure that the level of backlog per month is less than the requirement imposed by the management. In this section an analysis was made to assess the impact of the service level constraint on the solution. In particular we analyzed the range from 85% to 98% service level. For this analysis a model with six periods was used in Model-I to obtain the data of Table 7.

Service level (%)	**Δ-EV (%)**	**EV**	**Δ-WS (%)**	**Δ-EVPI (%)**
85	-0.0049549	1778265	-0.00845546	0.02481515
86	-0.004728278	1778670	-0.00767313	0.02031491
87	-0.004426116	1779210	-0.00632611	0.01172818
88	0.000383298	1787805	-0.003275	0.03149372
89	0.00060992	1788210	-0.0012282	0.01623799
90* (base case)	0	1787120	0	0
91	0.001138704	1789155	0.00185731	-0.00497622
92	0.001440866	1789695	0.0035389	-0.01640499
93	0.001667487	1790100	0.00469581	-0.0240855
94	0.011249944	1807225	0.00685767	0.04860283
95	0.009569027	1804221	0.00874937	0.01653688
96	0.01269081	1809800	0.00976808	0.0375474
97	0.012917431	1810205	0.01147531	0.02518104
98	0.013231904	1810767	0.01331573	0.01251793

Table 7. Impact of the service level constraint on the EV, WS and EVPI indicator

From Table 7 it can be seen that when the service level decreases, the indicator Expected Value reduces. The EV indicator gradually increases as the service level increases. The same pattern is observe for indicators WS and EVPI. We complement the analysis with the results presented in Table 8, where it is clear that the service level constraint significantly affects the decision variables. In particular, we note that as the service level increases, more workers are assigned to production and with this measure, the company produces more units and this also increases the level of inventory. With these countermeasures, it is logical to observe that backlog in the sixth period reduces.

Service level (%)	P_1	R_1	F_6	X_1	I_1	S_6
86	32	0	0	320	170	47
90	32	1	0	320	170	33
94	32	1	0	320	170	20
98	33	0	0	330	180	6

Table 8. Impact of the service level constraint on the decision variables

A company interested in the application of this type of models, must begin with the identification of the stochastic variable, then collection of data is needed in order to determine a probability distribution function that will serve to model such uncertainty. A decision tree is required to define the set of scenarios that will be generated in the optimization.

7. - CONCLUSION

This paper develops two multi-stage stochastic models for the production planning of a furniture company. The stochastic variable considered in the study is the production capacity. The models determine for each period, the level of the workforce, inventory, production and backlog. Discrete and continuous probability functions were tested and used as a base for the development of the optimization models. We noted that as more periods were considered in the optimization, the size of the models in terms of the number of variables and constraints increased. Also we noted that Model-I that uses the continuous distribution required more computational time and iterations to obtain a solution. From the obtained results, we observed that the development of the stochastic models is justified since the EVPI indicator was considerable especially when the number of periods increases. This EVPI indicator denotes the economic benefit that the company may obtain with the stochastic models. In the sensitivity analysis section, it was observed how the model adjusted certain variables in response to the variation of several costs. Additionally, we noted that as the service level increases, more workers, production and inventory are needed and less backlog is permitted. As future work, a more realistic model can be developed, where two random variables, namely the production capacity and the demand can be considered. As a recommendation we included the link: https://doi.org/10.1155/2013/949131, where future trends and uses of stochastic optimization can be found in many kinds of industrial, biological, engineering and economic problems.

REFERENCES

[1] M. Nowak, "An interactive procedure for aggregate production planning", *Croatian Operational Research Review*, vol. 4, no. 1, pp. 247–257, 2013.

[2] A. Jamalnia, J.B. Yang, D.L. Xu and A. Feili, "Novel decision model based on mixed chase and level strategy for aggregate production planning under uncertainty: Case study in beverage industry", *Computers & Industrial Engineering*, vol. 114, pp. 54-68, 2017.

[3] H. Zhao, E. Huang, R. Dou and K. Wu, "A multi-objective production planning problem with the consideration of time and cost in clinical trials", *Expert Systems with Applications*, vol. 124, pp. 25-38, 2019.

[4] T.R. Rakes, L.S. Franz, and A.J. Wynne, "Aggregate production planning using chance-constrained goal programming", *International Journal of Production Research*, vol. 22, no. 4, pp. 673–684, 1984.

[5] Y.K. Chen and H.C. Liao, "An investigation on selection of simplified aggregate production planning strategies using MADM approaches". *International Journal of Production Research*, vol. 41, no. 14, pp. 3359–3374, 2003.

[6] J.R. Birge and F. Louveaux, "Introduction to Stochastic Programming". Springer, New York, NY, 1997.

[7] H.M. Wagner and T.M. Whitin, "Dynamic version of the economic lot size model", *Management Science*, vol. 5, no. 12, pp. 1763–1893, 2004.

[8] M.K. Zanjani, M. Nourelfath, D. Ait-Kadi, "A scenario decomposition approach for stochastic production planning in sawmills", *Journal of the Operational Research Society,* vol. 64, no. 1, pp. 48–59, 2013.

[9] K. Huang, "Multi-stage stochastic programming models for production planning", Ph.D. Dissertation, School of Industrial and Systems Engineering, Georgia Institute of Technology, 2005.

[10] S. M. J. Mirzapour Al-e-hashem, A. Baboli and Z. Sazvar, "A stochastic aggregate production planning model in a green supply chain: Considering flexible lead times, nonlinear purchase and shortage cost functions", *European Journal of Operational Research*, vol. 230, no. 1, pp. 26–41, 2013.

[11] G. R. Nasiri, R. Zolfaghari, H. Davoudpour, "An integrated supply chain production-distribution planning with stochastic demands", *Computers & Industrial Engineering*, vol. 77, pp. 35-45, 2014.

[12] G. R., Nasiri, H. Davoudpour and B. Karimi, "The impact of integrated analysis on supply chain management: A coordinated approach for inventory control policy", *Supply Chain Management*, vol. 15, no. 4, pp. 277–289, 2010a.

[13] G.R., Nasiri, H. Davoudpour and Y. Movahedi, "A genetic algorithm approach for the multi-commodity, multi-period distribution planning in a supply chain network design", *Swarm, Evolutionary, and Memetic Computing, SEMCCO 2010, Lecture Notes in Computer Science*, vol. 6466, Springer, Berlin, Heidelberg.

[14] Y. Ning, J. Liu and L. Yan, "Uncertain aggregate production planning", *Soft Computing*, vol. 17, no. 4, pp. 617–624, 2013.

[15] K. Lieckens and N. Vandaele, "A decision support system for the stochastic aggregate planning problem", 2014. Available at SSRN: http://ssrn.com/abstract=2419376/16/03/20.

[16] S. C. H. Leung and Y. Wu, "A robust optimization model for stochastic aggregate production planning", *Production Planning & Control*, vol. 15, no, 5, pp. 502–514, 2004.

[17] A. P. Kanyalkar and G.K. Adil, "A robust optimization model for aggregate and detailed planning of a multi-site procurement-production-distribution system", *International Journal of Production Research*, vol. 48, no. 3, pp. 635–656, 2010.

[18] S. M. J. Mirzapour Al-e-hashem, H. Malekly, M.B. Aryanezhad, "A multiobjective robust optimization model for multi-product multi-site aggregate production planning in a supply chain under uncertainty", *International Journal of Production Economics*, vol. 134, no. 1, pp. 28–42, 2011.

[19] S. M. J. Mirzapour Al-e-hashem, M.B. Aryanezhad and S.J. Sadjadi, "An efficient algorithm to solve a multi-objective robust aggregate production planning in an uncertain environment", *The International Journal of Advanced Manufacturing Technology*, vol. 58, no. 5, pp. 765–782, 2012.

[20] A. Makui, M. Heydari, A. Aazami and E. Dehghani, “Accelerating Benders decomposition approach for robust aggregate production planning of products with a very limited expiration date”, *Computers & Industrial Engineering*, vol. 100, pp. 34–51, 2016.
[21] Z. Hu and G. Hu, “A multi-stage stochastic programming for lot-sizing and scheduling under demand uncertainty”, *Computers & Industrial Engineering*, vol. 119, pp. 157-166, 2018.
[22] E. Körpeoglu, H. Yaman, and M.S. Aktürk, “A multi-stage stochastic programming approach in master production scheduling”, *European Journal of Operational Research*, vol. 213, no. 1, pp. 166-179, 2011.