International Journal of Modern Physics C
Vol. 21, No. 7 (2010) 931–942

DOI: 10.1142/S0129183110015579

World Scientific
www.worldscientific.com

# CONSERVATION FEATURES IN BINARY COLLISIONS FOR RULE 110 CELLULAR AUTOMATON

JOSE M. SAUSEDO-SOLORIO

*Centro de Investigación Avanzada en Ingeniería Industrial*
*Universidad Autónoma del Estado de Hidalgo*
*Cd. Universitaria, Pachuca, Hidalgo 42184, México*
*sausedo@uaeh.edu.mx*

Received 7 December 2009
Accepted 10 June 2010

This paper investigates collisions of gliders generated by one-dimensional Rule 110 cellular automaton. A specific value associated with each glider and an algebraic equation that describes the collision between two gliders were found. Because the products of the collision between two gliders may result in no gliders or one, two or more gliders, this equation states that the total sum of the associated values corresponding to colliding gliders equals the sum of the values of the gliders which are products of the collision. Moreover, an analogy is proposed between the glider collisions and the collisions of physical particles with the equation corresponding to colliding gliders being similar to the equation of energy conservation in physics. In this scheme, even without carrying out the temporal evolution for a collision, it can be determined if a possible combination of resulting gliders accomplishes the equation corresponding to that collision.

*Keywords*: Nonlinear dynamical systems; particle collision; cellular automata; Rule 110; gliders; conservation equation.

PACS Nos.: 45.20.df; 89.20.Ff; 89.75.Fb.

## 1. Introduction

In recent years, cellular automata (CA) have gained attention by proving their capacity for analyzing complex systems, generating new concepts and even their application to physical systems. Hence the characterization of these types of systems is very important, with specific examples of CA applications being[1]: the characterization of complex dynamic systems based on statistical properties, proving criteria for self-organization using statistical complexity in models of excitable media and the behavior of physical systems without taking into account small-scale details.[2,3] Also, there have been reports of particle-like objects that propagate in several spatially-extended dynamic systems and interact among them.[4] In particular, the one-dimensional Rule 110 Cellular Automaton[a] has been widely studied in the last decade because of its capacity to produce universal complex behaviors; however a cell takes into account the actual state of just three neighborhoods and each cell has only two states.[5] Moreover, it was first conjectured by Wolfram that this cellular automaton may be universal. This statement was proved by Cooks implementing a cyclic tag system using Rule 110.[4,5]

A distinctive feature of the Rule 110 is the formation of a periodic background in space and time which is called *ether*. In conjunction with this regular mosaic, other structures known as *gliders* are formed as time evolves. Such gliders move with constant lateral displacement. However, such displacement may be different between one glider and another, resulting in collisions between them.

Collisions may yield other or even the same combinations of gliders also called *products* here. This feature has been studied to obtain both a theoretical understanding of this behavior and implement unconventional computer systems.[6,7]

Most of the work of Rule 110 has been done from the perspective of Computer Theory or from using Complex Systems analysis.[8] Moreover, there are previous research findings for Rule 110 with a general scope, which have considered algebraic features of cellular automata to provide invariant attributes in the sense of group theory.[9,10] Collisions among gliders have been analyzed by controlling their relative period as a way of producing them more easily.[11] However, to date, there has been a lack of published research on the characterization of gliders in this cellular automaton, which considers them as interacting objects.

In general according to Rule 110, gliders may be generated with specific initial conditions or as products of collisions with other gliders.[5,12] However, in this work, all gliders used have been created from initial conditions and are considered to be particles with a fixed trajectory provided they do not collide with other gliders. The aim of this paper is to state a quantitative characterization of the structures from the Rule 110 and to establish relations derived from collisions among gliders as an analogy with collisions of physical particles. The analysis is based on computational experimentation by causing two gliders to collide and observing the products of the collision.

The remainder of the paper is organized as follows: Sec. 2 is devoted to exposing the basic concepts of one-dimensional cellular automata. Section 3 explains how collisions among gliders are expressed in terms of algebraic equations. Section 4 provides the values for found constants as well as their interpretation. Section 5 states the conclusions reached about the utility of the constant associated with each glider.

## 2. Basic Concepts

In general, cellular automata are defined by means of a tuple $\{\Sigma, r, \phi, C\}$, where $\Sigma$ is a finite set of allowed states for each cell, $r \in \mathbb{Z}^+$ is the number of neighbors

[a]From here onwards, we will just use "Rule 110" to refer to Rule 110 Cellular Automaton.

with respect to each side of a cell, $\phi : \Sigma^{2r+1} \to \Sigma$ is the evolution rule determining the next state for every cell as a function of its own state and the states of its $2r$ neighboring cells at current time, and $C : \mathbb{Z}_m \to \Sigma$ is the initial configuration, $\mathbb{Z}_m = \{0, \ldots, m-1\}$, and $m \in \mathbb{Z}^+$ is the size of $C$. Hence, $C$ contains the initial state of every cell at the starting time of the evolution. In this way, CA are dynamic systems, not only with discrete spatial domain, but also with discrete temporal domain, where their spatial evolution is carried out through interactions with their nearest neighbors.

The particular case being analyzed is a cellular automaton whose rule of evolution is the Rule 110 defined in Table 1. For this rule, the set of states is $\Sigma = \{0, 1\}$ and $r = 1$ (a single neighbor to each side of the cell), therefore an initial configuration may be specified by a one-dimensional finite chain of 0's and 1's. A particular evolution can be seen in Fig. 1.

In this cellular automaton there are 14 known individual gliders represented by the set $M = \{A, B, \bar{B}, \bar{B}_8, C_1, C_2, C_3, D_1, D_2, E, \bar{E}, F, G, H\}$ plus a glider called *Gun*, which produces several of the gliders of $M$ as time evolves. So the glider *Gun* is not considered an individual structure.[5,6] Here, we consider only binary

Table 1. Evolution for cellular automaton based on Rule 110.

| Neighborhood | Evolution | Neighborhood | Evolution |
|---|---|---|---|
| 000 | 0 | 100 | 0 |
| 001 | 1 | 101 | 1 |
| 010 | 1 | 110 | 1 |
| 011 | 1 | 111 | 0 |

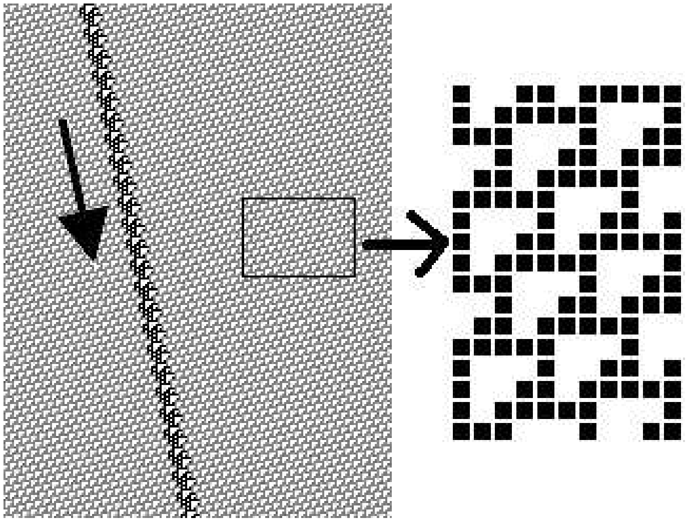


Fig. 1. A typical glider (darker structure) moving through ether (light gray color). Temporal evolution follows the downward direction. The detail shows ether structure.

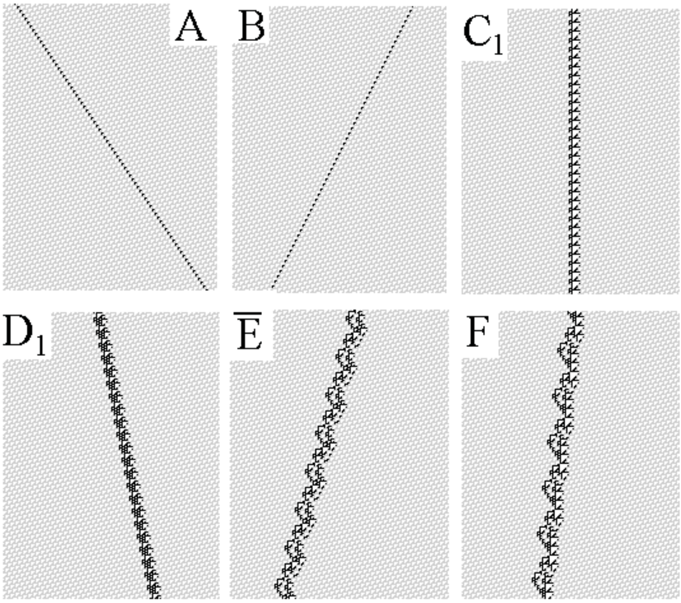



Fig. 2. 6 of the 14 gliders defined in $M$ traveling downward in ether; each is labeled with its own name.

collisions among gliders belonging to $M$. Some elements of this set are shown in Fig. 2, however, a complete list can be found in the McIntosh catalog.[13]

In order to generate ether as well as a specific glider, it is necessary to choose the appropriate initial conditions for the beginning of the evolution. In particular, the ether in Rule 110 is generated by a sequence consisting of 14 cells. In the case of gliders, the length of the sequence is variable, for example, to generate gliders $A$, $C_1$ and $E$, the lengths of the ships are 6, 23 and 29 cells, respectively. It is possible to generate gliders with more than one set of initial conditions[b]; for example, glider $A$ can be generated with the following two sequences (phases) of 6 cells: 111110, and 100011.[6]

Most of the possible combinations[c] of binary collisions among gliders of $M$ have been studied and classified previously in atlases and catalogs.[4,5,13] However, up to now, there has not been a collision-based analysis that provides features useful for exploring the underlying quantitative properties from the interaction among gliders.

## 3. Quantitative Relations in Binary Collisions of Gliders

A schematic representation of a collision is shown in Fig. 3, where initial gliders $\mu_i$ and $\mu_j$ collide to produce the gliders labeled as $\mu_1$, $\mu_2$, $\mu_3$ and so on, appearing at

[b]In the terminology of cellular automata, it is called *phase* to each of those sequences.
[c]Although it is highlighted below, a collision is not found in catalogs yet.

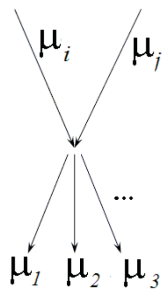



Fig. 3. Schematic representation of a binary collision between two gliders $\mu_i$ and $\mu_j$. The resulting products are labeled as $\mu_1$, $\mu_2$, and $\mu_3$.

the bottom of the figure. A collision may generate no gliders, one or several gliders as its *products.*

Equation (1) highlights the notation used to specify a collision between two incident gliders. Labels on the left-hand side correspond to the colliding gliders, whereas labels on the right-hand side indicate the products. Here, this equation is called the *production relation.*

$$\mu_i \oplus \mu_j \rightarrow \mu_1 + \cdots + \mu_n \tag{1}$$

where $\mu_k \in M$ and $k \in \mathbb{N}$. Symbol $\oplus$ indicates the interaction (collision) between gliders $\mu_i$ and $\mu_j$, while the plus sign (+) represents the collection[d] of resulting products labeled as $\mu_1, \mu_2, \ldots, \mu_n$.

With 14 gliders in $M$, there are 91 ($14 \times 13/2$) possible results of binary collisions, because for two gliders $\mu_1, \mu_2 \in M$ under this notation, the result of $\mu_1 \oplus \mu_2$ is the same as for $\mu_2 \oplus \mu_1$. Furthermore, some gliders have the same *horizontal speed*, meaning that they travel in parallel, hence they can never collide. The subsets of gliders that move in parallel are $\{C_1, C_2, C_3\}$, $\{B, \bar{B}, \bar{B}_8\}$, and $\{D_1, D_2\}$. Additionally, there are some *soliton-like* binary collisions, i.e. interactions in which at least one of the gliders remains without change after collision. An example of this type of collision process is[e] $A \oplus \bar{E} \rightarrow A + \bar{E}$.

A typical example of a binary collision can be observed in Fig. 4, where gliders $C_1$ and $\bar{B}$ collide, yielding as products two[f] $B$ gliders and one $F$ glider ($\bar{B} \oplus C_1 \rightarrow 2B + F$) and the corresponding algebraic equation proposed can be written as $\xi_{\bar{B}} + \xi_{C_1} = 2\xi_B + \xi_F$.

[d]Here, we use the sign of summation, instead of a comma as in the symbology of set theory.
[e]Hereafter the symbols corresponding to gliders on both sides of a production relation are written in alphabetical order, regardless of its position when a collision is observed in the time evolution graph.
[f]To verify that the structure labeled with $2B$ actually consists of two $B$ particles, there are two ways to proceed: One, by performing an amplification of the figure and comparing with the corresponding *tilling* of glider B.[13] Two, by causing a collision of a known particle (i.e. $A$) with this structure and observing that one of the $B$ particles is eliminated with $A$ ($A \oplus 2B \rightarrow B$), while the remaining $B$ glider continues its path without change.

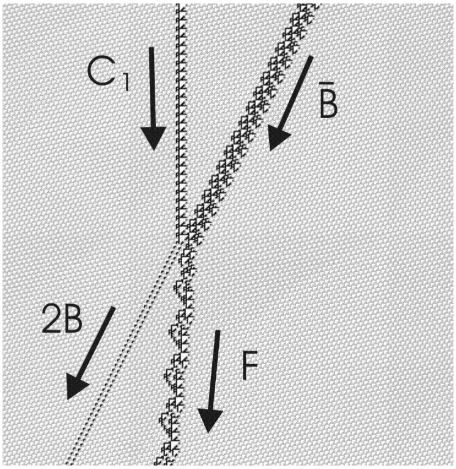



Fig. 4. The binary collision between gliders $C_1$ and $\bar{B}$ during a productive relation denoted by $C_1 \oplus \bar{B} \rightarrow 2B + F$. The corresponding algebraic equation proposed is $\xi_{C_1} + \xi_{\bar{B}} = 2\xi_B + \xi_F$.

After consideration, a total of 83 pairs of colliding gliders can be listed which are represented as production relations in Table 2. In this table, the $\Phi$ symbol is used to denote that no particle is obtained after a collision, being its *associated constant* equal to zero ($\xi_\Phi = 0$).

Most pairs of colliding particles can be found to collide in more than one way. For example, the collision $A \oplus \bar{B}_8 \rightarrow C_2$ listed in Table 2, it can also be found as $A \oplus \bar{B}_8 \rightarrow 4A + \bar{E}$. Thus, it is possible at times for collisions to result in more than one combination of particles. In Table 2, only one possibility of these combinations has been written for each pair, the rest can be found elsewhere.[13] In order to get different results from the collision of a pair of gliders, they must collide with a different contact point or relative phase. This is achieved by generating the gliders with different initial conditions.[g]

## 4. Results

In general, for the production relation in Eq. (1), the following corresponding algebraic equation is proposed:

$$\xi_{\mu_i} + \xi_{\mu_j} = \xi_{\mu_1} + \cdots + \xi_{\mu_n} \tag{2}$$

where each $\xi_{\mu_k}$ is the unknown glider constant associated to glider $\mu_k$. In both sides of this equation, the plus sign means algebraic summation of the constants, so for each production relation in Table 2, it is possible to write a corresponding linear algebraic equation involving unknowns $\xi_\mu$. All of these algebraic equations form a

[g] The ways that a glider can be generated are called *Periods*, which are used to control the contact point at the time that two gliders collide.

Table 2. Production relations corresponding to collisions between two gliders.

| | | | |
|---|---|---|---|
| $A \oplus B \rightarrow \Phi$ | $A \oplus \bar{B} \rightarrow \Phi$ | $A \oplus \bar{B}_8 \rightarrow C_2$ | $A \oplus C_1 \rightarrow F$ |
| $A \oplus C_2 \rightarrow C_1$ | $A \oplus C_3 \rightarrow C_2$ | $A \oplus D_1 \rightarrow C_2$ | $A \oplus D_2 \rightarrow D_1$ |
| $A \oplus E \rightarrow D_1$ | $A \oplus \bar{E} \rightarrow A + \bar{E}$ | $A \oplus F \rightarrow 4B + C_2$ | $A \oplus G \rightarrow \bar{E} + C_1$ |
| $A \oplus H \rightarrow C_2$ | $B \oplus C_1 \rightarrow C_2$ | $B \oplus C_2 \rightarrow D_1$ | $B \oplus C_3 \rightarrow E$ |
| $B \oplus D_1 \rightarrow E$ | $B \oplus D_2 \rightarrow A + \bar{E}$ | $B \oplus E \rightarrow E_2$ | $B \oplus \bar{E} \rightarrow 2A + 3B + \bar{E}$ |
| $B \oplus F \rightarrow 2A + D_1$ | $B \oplus G \rightarrow G_2$ | $B \oplus H \rightarrow \bar{E}$ | $\bar{B} \oplus C_1 \rightarrow 2B + F$ |
| $\bar{B} \oplus C_2 \rightarrow 3A + \bar{E}$ | $\bar{B} \oplus C_3 \rightarrow 2A + \bar{E}$ | $\bar{B} \oplus D_1 \rightarrow E$ | $\bar{B} \oplus D_2 \rightarrow A + \bar{E}$ |
| $\bar{B} \oplus E \rightarrow A + 4B + C_2$ | $\bar{B} \oplus \bar{E} \rightarrow 4A + 5B + \bar{E}$ | $\bar{B} \oplus F \rightarrow A + 4A + \bar{E}$ | $\bar{B} \oplus G \rightarrow 4B$ |
| $\bar{B} \oplus H \rightarrow A + 2C_2 + \bar{E}$ | $\bar{B}_8 \oplus C_1 \rightarrow A + 2\bar{E}$ | $\bar{B}_8 \oplus C_2 \rightarrow 2A + 3B + 2C_2$ | $\bar{B}_8 \oplus C_3 \rightarrow 2A + 3B$ |
| $\bar{B}_8 \oplus D_1 \rightarrow A + B + \bar{B}$ | $\bar{B}_8 \oplus D_2 \rightarrow 2A + 4B$ | $\bar{B}_8 \oplus E \rightarrow 2A + B + G$ | $\bar{B}_8 \oplus \bar{E} \rightarrow 3A + 2\bar{E}$ |
| $\bar{B}_8 \oplus F \rightarrow 2A + 2B + C_2 + \bar{B} + F$ | $\bar{B}_8 \oplus G \rightarrow 4A + 4B + \bar{E}$ | $\bar{B}_8 \oplus H \rightarrow 2A + C_2 + \bar{E}$ | $C_1 \oplus D_1 \rightarrow 4A + 3B$ |
| $C_1 \oplus D_2 \rightarrow 2A + 2B$ | $C_1 \oplus E \rightarrow A + \bar{E} + F$ | $C_1 \oplus \bar{E} \rightarrow C_1 + \bar{E}$ | $C_1 \oplus F \rightarrow C_1 + F$ |
| $C_1 \oplus G \rightarrow 3A + F$ | $C_1 \oplus H \rightarrow 3A + 3B + 2C_2$ | $C_2 \oplus D_1 \rightarrow 2A + 2B$ | $C_2 \oplus D_2 \rightarrow A + 2B$ |
| $C_2 \oplus E \rightarrow A + 2B$ | $C_2 \oplus \bar{E} \rightarrow 3B$ | $C_2 \oplus F \rightarrow C_1 + \bar{B} + F$ | $C_2 \oplus G \rightarrow 3B + C_2$ |
| $C_2 \oplus H \rightarrow 3A + 2B + \bar{B}$ | $C_3 \oplus D_1 \rightarrow A + 2B$ | $C_3 \oplus D_2 \rightarrow A + 3B$ | $C_3 \oplus E \rightarrow A + G$ |
| $C_3 \oplus \bar{E} \rightarrow 2C_1$ | $C_3 \oplus F \rightarrow C_1 + C_2$ | $C_3 \oplus G \rightarrow \bar{E}$ | $C_3 \oplus H \rightarrow 2B + \bar{B} + C_3 + F$ |
| $D_1 \oplus E \rightarrow 2B$ | $D_1 \oplus \bar{E} \rightarrow 4B$ | $D_1 \oplus F \rightarrow 2A$ | $D_1 \oplus G \rightarrow \bar{E}$ |
| $D_1 \oplus H \rightarrow A + D_1 + E$ | $D_2 \oplus E \rightarrow G$ | $D_2 \oplus \bar{E} \rightarrow 5B$ | $D_2 \oplus F \rightarrow 2A + B$ |
| $D_2 \oplus G \rightarrow A + 3B + C_2 + G$ | $D_2 \oplus H \rightarrow C_1 + \bar{E}$ | $E \oplus F \rightarrow 4A + 3B$ | $E \oplus G \rightarrow F$ |
| $E \oplus H \rightarrow 5A + 2\bar{E}$ | $\bar{E} \oplus F \rightarrow B$ | $\bar{E} \oplus G \rightarrow 4A + \bar{E}$ | $\bar{E} \oplus H \rightarrow A + 3B + C_1 + \bar{E}$ |
| $F \oplus G \rightarrow 3A + \bar{E}$ | $F \oplus H \rightarrow F + D_1$ | $G \oplus H \rightarrow A + 2\bar{E} + F$ | |

Table 3. Values of the unknowns found for each glider of the set $M$.

| | | | | |
|---|---|---|---|---|
| $\xi_\Phi = 0$ | $\xi_A = 2$ | $\xi_B = -2$ | $\xi_{\bar{B}} = -2$ | $\xi_{\bar{B}_8} = -1$ |
| $\xi_{C_1} = 3$ | $\xi_{C_2} = 1$ | $\xi_{C_3} = -1$ | $\xi_{D_1} = -1$ | $\xi_{D_2} = -3$ |
| $\xi_E = -3$ | $\xi_{\bar{E}} = -7$ | $\xi_F = 5$ | $\xi_G = -6$ | $\xi_H = -1$ |

system of linear equations. Since the system has more equations than variables, this is an over-determined system. The full solution was obtained by a trial-and-error procedure, assigning an arbitrary value[h] to one $\xi_\mu$, and then obtaining the values for the rest of the variables. The resulting values obtained for each unknown are shown in Table 3. For example, for the collision in Fig. 4, by replacing values from Table 3 in the corresponding algebraic equation $\xi_{C_1}+\xi_{\bar{B}} = 2\xi_B+\xi_F$, we get $3+(-2)$ on the left-hand side and $2(-2) + 5$ on the right-hand side, ensuring equality.

The following physical analogy for the collision of two gliders is proposed. For one dimension, the energy of a physical particle is a signed quantity. This quantity is conserved[i] during the elastic collision of two particles. In this way, we can consider each value of $\xi_\mu$ as if it were the energy of the "particle" $\mu$. Then the algebraic equation governing the collision of gliders corresponds to the conservation of energy for gliders in this CA. Regardless of this analogy, it must be emphasized that the origin of each $\xi_\mu$ is geometric and it is related to a shift in the ether on the right-hand side of the traveling glider.

Included in Table 2, as an additional result is the collision $B \oplus F \to 2A + D_1$ not previously catalogued in any atlas for Rule 110.[13] Also two structures are not included in the set $M$: $E_2$ and $G_2$ (not catalogued as individual gliders), appearing as collision products in $B \oplus E \to E_2$ and $B \oplus G \to G_2$. Moreover, the values $E_2 = -5$ and $G_2 = -8$ have been found for these structures, both consistent with the system of linear algebraic equations proposed. That table shows 83 collisions, for 80 of them it is possible to write an algebraic equation in the form of Eq. (2). But for the remaining three ($A \oplus F \to 4B + C_2$, $E \oplus G \to F$, and $\bar{E} \oplus G \to 4A + \bar{E}$), no set of gliders has been found or reported whose constants fulfill the corresponding algebraic equations. For those cases it is proposed here a constant[j] $\alpha = 14$ to include in each equation in the following form

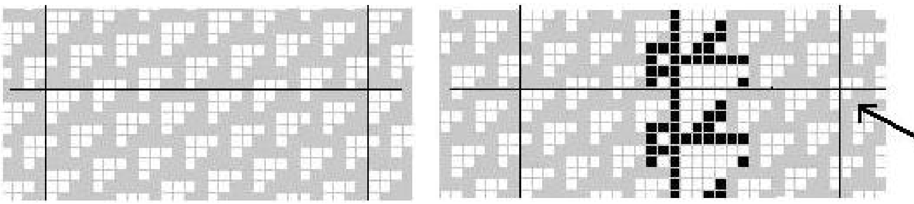
$$\begin{aligned} \xi_A + \xi_F &= 4\xi_B + \xi_{C_2} + \alpha\,, \\ \xi_E + \xi_G &= \xi_F - \alpha\,, \\ \xi_{\bar{E}} + \xi_G &= 4\xi_A + \xi_{\bar{E}} - \alpha\,. \end{aligned} \tag{3}$$

As a step in the process of solving this problem, we have determined that the respective balance equations are fulfilled if such constant is added to or subtracted

[h]It is necessary to apply the constraint of $\xi_A + \xi_B = 0$ in order that the numeric solution be self-consistent.

[i]This general law is called *Conservation Law of Energy*.

[j]Here, it is considered $\alpha$ as a *Structure Constant* due to its origin from the ether structure.



(a) (b)

Fig. 5. Comparison of the displacement produced in ether by a traveling glider. Intersections of horizontal and vertical lines serve as references to observe the shift on ether. On the left, ether is depicted alone. On the right, glider $C_1$ travels in a downward motion. The arrow points to the intersection which demonstrates the ether's shift.

from each of these equations. In Eq. (3), the value of $\alpha$ is equal to the length of the sequence that generates ether. In fact, the whole algebraic system has an infinite number of solutions shown in Table 3, is just one particular solution.

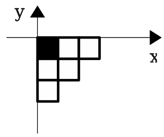
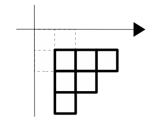
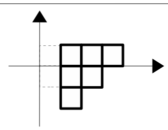
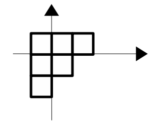
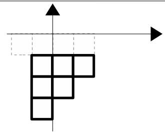
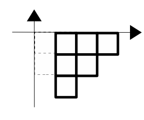
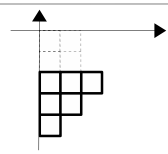
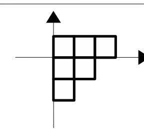
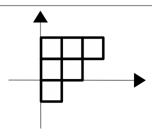
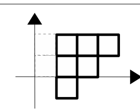
Whenever a glider goes through ether, the ether pattern has a relative displacement of one side of the glider with respect to the other side. Figure 5 shows how such a displacement can be detected. In this figure there are three reference lines superimposed on the ether pattern. Two of the lines are vertical and one is horizontal. In Fig. 5(a), the two intersections of the line match the ether pattern at the same relative point; whereas in Fig. 5(b) as glider $C_1$ travels down through the ether, there is a shift in the ether pattern to the right of the glider. It can be seen as indicated by the arrow, that the ether pattern to the right of the figure is no longer in the same position as before, relative to the intersection of the corresponding lines. Such displacement depends on each glider. Moreover, if the two gliders have the same $\xi_\mu$, the ether experiences the same shift.

Table 4 shows the displacements $(\delta x, \delta y)$ of each glider $\mu$, the ether displacement graph and the corresponding constant $\xi_\mu$. The same cell was taken as reference per Fig. 5(a), and the displacement is counted by the number of cells with the same signs for distances as used in the Cartesian coordinate system. The column of constants $\xi_\mu$ taken from the set of solutions already encountered by each glider $1, -1, 2, -2, 3, -3, 5, -6, -7$ in Table 3. The gliders with the same value $\xi$, displace the ether in the same magnitude and direction and are found in the table on the same line.

This table shows only one of an infinite number of ether displacements. Because the ether pattern is periodic, a displacement, as an example, $(\delta x = +4, \delta y = +1)$, takes the reference cell to the same location as with $(\delta x = +2, \delta y = -3)$.

Displacements behave in the same way as the components of a two-dimensional vector. As an example, from Table 4, for glider $C_1$ (with $\xi_{C_1} = +3$) the cell of reference undergoes a displacement of $(\delta x_1 = -1, \delta y_1 = +1)$, and for $D_2$ (with $\xi_{D_2} = -3$) a displacement of $(\delta x_2 = +1, \delta y_2 = -1)$. If we cause $C_1$ to collide with $D_2$, the rule of production for this collision and the corresponding algebraic

Table 4. Displacement of the ether pattern and values associated with each glider.

| $\mu$ | $(\delta x, \delta y)$ | Ether pattern | $\xi_\mu$ | $\mu$ | $(\delta x, \delta y)$ | Ether pattern | $\xi_\mu$ |
|---|---|---|---|---|---|---|---|
| Reference | (0, 0) |  | 0 | $D_2$, $E$ | (1, -1) |  | -3 |
| $\overline{B}_8$, $C_3$, $D_1$, $H$ | (1, 1) |  | -1 | $C_1$ | (-1, 1) |  | +3 |
| $C_2$ | (-1, -1) |  | +1 | $F$ | (1, -1) |  | +5 |
| $B$, $\overline{B}$ | (0, -2) |  | -2 | $G$ | (0, 1) |  | -6 |
| $A$ | (0, 2) |  | +2 | $\overline{E}$ | (1, 2) |  | -7 |

equation are $C_1 \oplus D_2 \rightarrow 2A + 2B$, and $\xi_{C_1} + \xi_{D_2} = 2\xi_A + 2\xi_B$. By replacing the values of every $\xi$ on the last equation, $(+3)+(-3) = 2(2)+2(-2)$, resulting in $0 \equiv 0$. Observe that the summation of the displacements of the initial gliders vanishes to $[(+3) + (-3) = 0]$. Because values cancel on both sides of the algebraic equation, it is expected that ether does not suffer any displacement. This can be seen in Fig. 6.

In this way, displacements $(\delta x, \delta y)$ behave in an arithmetical manner consistent with the $\xi_\mu$ associated with glider $\mu$. This is the way in which each displacement connects with its corresponding glider constant.

Although previously mentioned that the origin of the constants associated with each glider is of a geometric character, there is no metric discovered yet to establish a formal relationship between a displacement $(\delta x_\mu, \delta y_\mu)$ and the corresponding $\xi_\mu$ in the form $F(\delta x_\mu, \delta y_\mu) \sim f(\xi_\mu)$.

## 5. Concluding Remarks

In this paper, we suggest that:

- For each glider where $\mu \in M$, there is a quantity $\xi_\mu$ which fulfills a balanced equation.

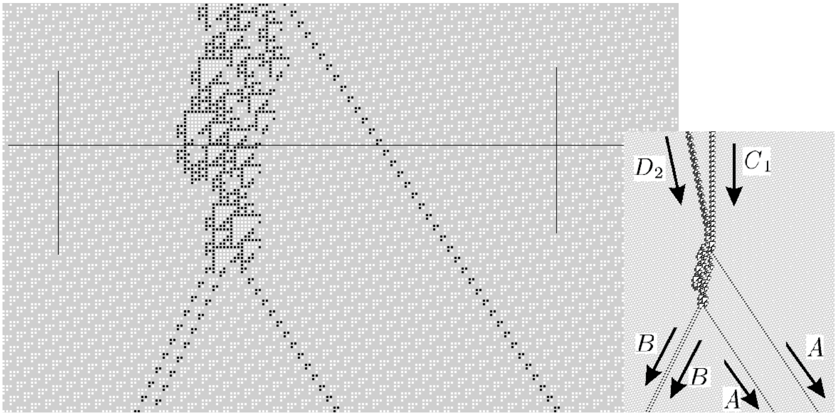



Fig. 6. Collision between gliders $C_1$ and $D_2$, the evolution is in accordance with the rule of production: $C_1 \oplus D_2 \rightarrow 2A + 2B$ as seen completely in the box. In the enlarged part of the figure, with the help of the previously calibrated lines, one can observe that the ether is not displaced.

- Each collision between two gliders corresponds to a balanced equation. This is a linear algebraic relationship with the unknowns being the $\xi_\mu$'s. On the left-hand side of the equation, the incident gliders are represented; on the right, the resulting gliders.
- The numeric quantity $\xi_\mu$ associated with each glider generated by Rule 110 represents a shift in the ether pattern.

An analogy was proposed for the interaction between two gliders with the collision of two physical particles. The algebraic equation in the gliders' case resembles the equation for the conservation of energy for physical particles. This is similar to several situation in physics where, for instance, energy, charge, etc. are scalar quantities conserved during a collision.

It is possible to use the value found for each glider and its related equations to construct an algebraic system valid for collisions among gliders. It is noteworthy that, with this tool and even without the temporal evolution, the manner in which the ether is displaced can be established, merely by knowing the gliders involved in the collision.

With respect to the balanced equations contained in Eq. (3), the addition or subtraction of a constant should not affect the validity of the solution seen in Table 3, because this is a particular solution. We consider the failure to find such sets of gliders does not detract from the usefulness of these results. We must wait for the necessary collisions to complete before this scheme can be found.

## Acknowledgments

The author wishes to express his gratitude to U.A.E.H. for the support given to realize this work; to G. Juarez-Martinez for his interest in this problem and his many useful suggestions and a special thanks to J. D. Seck Tuoh-Mora, for discussing the results and reviewing the entire manuscript.

## References

1. A. Shreim, P. Grassberger, W. Nadler, B. Samuelsson, J. E. S. Socolar and M. Paczuski, *Phys. Rev. Lett.* **98**, 198701 (2007).
2. C. R. Shalizi, K. L. Shalizi and R. Haslinger, *Phys. Rev. Lett.* **93**, 118701 (2004).
3. N. Israeli and N. Goldenfeld, *Phys. Rev. E* **73**, 026203 (2006).
4. S. Wolfram, *A New Kind of Science* (Wolfram Media, Illinois, 2002).
5. M. Cook, *Complex Systems* **15**, 1 (2004).
6. G. Juarez-Martinez, H. V. McIntosh and J. C. S. Tuoh-Mora, *Int. J. Unconv. Comp.* **21**, 1 (2006).
7. A. Adamatzky (ed.), *Collision-Based Computing* (Springer, New York, 2002).
8. L. Landau and E. M. Lifshitz, *Mechanics*, Vol. 1, 3rd edn. (Butterworth-Heinemann, Massachusetts, 1976).
9. M. Pivato, *Nonlinearity* **15**, 1781 (2002).
10. M. Pivato, *Ergod. Th. Dynam. Sys.* **26**, 1 (2006).
11. G. Juarez-Martinez, H. V. McIntosh and J. C. S. Tuoh-Mora, *Lect. Notes in Comp. Sci.* **2801**, 175 (2003).
12. X. B. Li, R. Jiang and Q. S. Wu, *Phys. Rev. E* **68**, 016117 (2003).
13. H. V. Mclntosh, http://delta.cs.cinvestav.mx/∼mcintosh/comun/summer2001/bookcollisionsHtml/bookcollisions.html.