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Guaranteed cost control of linear systems with distributed delays: A complete type functionals approach

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REFERENCES (21) EXPORT CITATION ABOUT

Abstract

The design of a control law guaranteeing an upper bound on the performance index of linear systems with point wise and distributed delay is addressed. The control law is computed through an iterative procedure: at each step, it is the solution that minimizes the Bellman type equation obtained from plugging in the functional of complete type associated to the closed loop system of the previous step. It is shown that at each step, the closed loop system remains into the class of systems with distributed delays, the stability of the closed loop is maintained, and the guaranteed cost does not increase. The allowed continuous time varying norm bounded uncertainties guaranteeing the cost are also characterized. An illustrative example validates the approach.

Keywords Complete type functionals - guaranteed cost control - suboptimal control - time delay systems

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Guaranteed Cost Control of Linear Systems with Distributed Delays: A Complete Type Functionals Approach

Omar Santos and Sabine Mondié

Abstract: The design of a control law guaranteeing an upper bound on the performance index of linear systems with point wise and distributed delay is addressed. The control law is computed through an iterative procedure: at each step, it is the solution that minimizes the Bellman type equation obtained from plugging in the functional of complete type associated to the closed loop system of the previous step. It is shown that at each step, the closed loop system remains into the class of systems with distributed delays, the stability of the closed loop is maintained, and the guaranteed cost does not increase. The allowed continuous time varying norm bounded uncertainties guaranteeing the cost are also characterized. An illustrative example validates the approach.

Keywords: Complete type functionals, guaranteed cost control, suboptimal control, time delay systems.

1. INTRODUCTION

The study of the linear quadratic optimal problem for time delay systems has a history of over 40 years. Many attempts to obtain a satisfactory solution have been made. One should mention the Maximum Principle [8], the suboptimal control where the terms with delays are considered as a disturbance [12], the use of operators in infinite dimensional spaces [1,10] and [19], the use of the Dynamic Programming Principle [9,17,18] and the recent contribution [16] where complete type Lyapunov Krasovskii functionals [7,15] are employed. Observe that these approaches are based on a set of partial differential equations, ordinary equations and algebraic equations that allow to compute the Bellman functional. This system of equations can be viewed as the counterpart of the classical matrix Riccati equation for the LQR problem for systems without delays. It should also be mentioned that, to the best of the author's knowledge, no results have been reported which guarantees the unicity of the solution of this set of partial differential equations. As a consequence, it makes sense to seek an upper bound for the performance index in the spirit of the Guaranteed Cost Control Approach [2,13,21].

Here, we achieve this goal through the iterative construction of the complete type Lyapunov Krasovskii functionals introduced in [7,15] into the Bellman type equation. We take advantage of the fact that in the complete type approach the derivative is prescribed; therefore we are able to use an analytic optimization tool in order to obtain the controller. The resulting control law belongs to the class of admissible control laws for the linear quadratic optimal control problem of time delay systems, characterized in the work of Krasovskii [9], thus allowing bounds on the performance index that are closer to the optimal. The complete type functional framework also permits to prove that the proposed iterative procedure satisfies at each step important properties: the system remains in the class of distributed time delay systems under consideration, the stability is preserved and the guaranteed cost control is not increased.

We consider time delay systems of the form

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-h) + \int_{-h}^{0} D(\theta) x(t+\theta) d\theta + Bu(t) \tag{1}$$

with $x(\zeta) = \varphi(\zeta)$, for $\zeta \in [-h, 0]$,

where $x(t) \in R^n$, $u(t) \in R^p$, $A_{0,1} \in R^{n\times n}$, $B \in R^{n\times p}$, $D(\theta) \in R^{n\times n}$, is defined for $\theta \in [-h, 0]$. Furthermore, each element of the matrix $D(\theta)$ is assumed to be continuous. The initial functions φ are bounded, piece-wise continuous with at most a finite number of discontinuity points.

Assumption A: A stabilizing controller of the form

$$u = \Gamma_0^{(0)} x(t) + \int_{-h}^{0} \Gamma_1^{(0)}(\theta) x(t+\theta) d\theta, \tag{2}$$

where $\Gamma_0^{(0)} \in R^{p\times n}$ and $\Gamma_1^{(0)}(\theta)$ is a continuous matrix function $p\times n$ defined on the interval $[-h, 0]$ for system (1) is known.

A number of possible design methods for this initial stabilizing control law are available in the literature: we would like to mention the contributions of [2,11] ($u = \Gamma_0^{(0)} x(t)$), of [14] ($u = G_1 x(t) + G_2 x(t-h)$, where G_1 and G_2 are constant matrices) and of [3] (distributed control

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