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Experimental results of a control time delay system using optimal control

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SUMMARY

The optimal control for a temperature system with time delay is considered. Experimental results of the control system are presented in this contribution. The integral term in the controller is approximated by a quadrature method. Experimental results obtained demonstrate the effectiveness of the approximation method. By a simple analysis in time domain, we demonstrate the robustness of the optimal controller. We compare the optimal control's performance with an industrial PID controller. This controller was robustly tuned. The experiments indicate the correct optimization of the plant when the optimal control was employed, despite limitations in the sensor, actuators, non-modeled dynamics, and uncertain parameters of the process. Copyright © 2011 John Wiley & Sons, Ltd.

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KEY WORDS: time delay systems; optimal control; experimental results; robust stability; industrial PID controller

1. INTRODUCTION

It is widely known that a number of time-delay systems are described by the following model:

$$\frac{Y(s)}{U(s)} = G(s) = \frac{K\mathrm{e}^{-hs}}{Ts+1} \tag{1}$$

Even though it might seem trivial to control stable systems described by the model of Equation (1), for which a PID controller may suffice, it is not so, as it is shown next. It is not straightforward to tune a PID controller for a first order and stable system with delays, because the presence of time delays in the input lead to either poor performance or unstable behavior [1–3]. In fact, specialized software exists for tuning PID controllers when an approximated model in the form given by Equation (1) is considered. This is the case of the Expert Tune Software, which has PID Loop Optimization for advanced tuning. Furthermore, numerous scientific reports have also been presented, indicating the complexity of using PID controllers for this class of plants. For example, costs are calculated when a PID controller is poorly tuned [4]. In [5] an approximated model of Equation (1) for a refining process is considered and some problems associated when PID badly tuned are reported. The same model and similar difficulties are reported in [6]. In contrast, we aim

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to develop an optimal control strategy, which we claim is more efficient than PID controllers. To validate our claims we also present a comparison study between an industrial PID (robustly tuned) and the optimal control law implemented by us.

The optimal control approach of time delay systems has been proposed as an alternative to PID controllers, namely the Maximum Principle [7], the suboptimal control (where time delays are considered as disturbances [8]), or the use of operators in infinite dimensional spaces [9–12]. Also, Dynamic Programming [13–16] and the use of complete type Lyapunov Krasovskii functionals [17] have been explored in order to obtain suboptimal control [18]. The considered systems in those reports present delays in the state, in the input or both. If a system displays delays in the state and the optimal control uses state feedback, then the control presents distributed delays in the states [13]. As it is proved in [19], when the system depicts delays in the control, the controller has distributed delays in the control signal. The implementation of control laws involving distributed delays has been studied by many authors. For example in [20, 21] the integral term of the controller is approximated by a sum of point-wise delays by using a numerical quadrature method. In [22] an optimal control for systems with delayed state is based on a numerical integration to solve the set of equations on a grid. The approximated solution is found step by step on each subinterval. This solution defines the matrices of the controller. Implementation problems have been reported regarding unstable delayed systems. In [23] a quadrature method is used in order to implement the integral term; however, when a more accurate approximation of the integral term is used, the closed loop system becomes unstable. In [24], an analysis of the causes of such behavior is presented: the characteristic equation in closed loop is a neutral type and the poles of large magnitude are unstable. The approximation problem is treated in the frequency domain considering a rectangular pulse in the integral and taking the approximation directly in the z-domain [25].

Despite the fact that stable systems are not thought to show the implementation problems of control laws of unstable systems [26], experimental validation, though important, is still missing. Moreover, this experimental validation shows that even in the presence of nonlinear disturbances and uncertain parameters, the optimal control law presents good performance.

Hence we present our own results for a temperature control system (stable in open-loop), which is regulated by an optimal control law of infinite horizon. The optimal control involves a distributed time delay in the controller and it is time invariant. We demonstrate that the exact control law is robustly stable under nonlinear disturbances. Our experimental results validate the affectivity of the quadrature approximation method.

Our results probe that implemented the optimal control law displays a similar performance, in terms of the Integral Square Error (ISE), than an industrial PID used for comparison purposes. Nevertheless, we must outline that the best performance of the PID was achieved when it was robustly tuned. Our experiments indicate that the optimal controller is more energy efficient, since it does not stay in the saturation region as much as the PID does.

The paper is organized as follows: In Section 2, considering nonlinear disturbances in the nominal model, robust stability conditions are obtained. Experimental results are presented in Section 3. Section 4 presents comparative results with an industrial PID controller and a different numerical integration method. Concluding remarks are given in Section 5.

Notation: Along the paper y_t denotes a piece of the trajectory of the considered system, i.e. $y_t = y(t+\theta)$, $\theta \in [-h, 0]$.

2. ROBUST STABILITY IN CLOSED LOOP

Consider the nominal system given previously by Equation (1). One can obtain the difference differential equation as

$$\begin{aligned} \dot{y}(t) &= -ay(t)+bu(t-h) \\ y(0) &= y_0, \quad u(\theta)=\mu(\theta), \quad \theta\in[-h,0] \end{aligned} \tag{2}$$

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where $a = 1/T$, $b = K/T$, and $\mu(\theta)$ is an arbitrary piecewise continuous function. We need to find a control law u such that the performance index

$$J = \int_0^{\infty} (y^2 Q + u^2 R) \mathrm{d}t \tag{3}$$

is minimum. Here $Q \geqslant 0$ and $R > 0$. In order to obtain the controller u, we use the approach given in [19], modified for the infinite horizon problem. Choosing $v(t) = u(t-h)$, the performance index can be rewritten as

$$J = \int_0^h y^2 Q \,\mathrm{d}t + \int_h^{\infty} (x^2 Q + v^2 R) \,\mathrm{d}t$$

and the system (2) is

$$\dot{y}(t) = -ay(t) + bv(t), \quad t \in [h, \infty)$$

with performance index

$$J = \int_h^{\infty} (x^2 Q + v^2 R) \mathrm{d}t$$

The control law $v(t)$ can be found by classical methods of delayed-free optimal control:

$$v(t) = -Fy(t)$$

where $F = -R^{-1} bP$, and P satisfies an algebraic Riccati equation [27]. As only $y(t-h)$ is available as feedback to the controller, then

$$u(t-h) = -F \mathrm{e}^{ah} y(t-h) - F \int_{t-h}^{t} \mathrm{e}^{-a(s-t)} bu(s-h) \mathrm{d}s \tag{4}$$

In Equation (4), the term $u(s-h)$ can be obtained by Equation (2), and the controller $u(t-h)$ can be rewritten as

$$u(t-h) = -F \mathrm{e}^{ah} y(t-h) - F \int_{t-h}^{t} \mathrm{e}^{-a(s-t)} [\dot{y}(s) + ay(s)] \mathrm{d}s$$

The first integral term is solved when integrating by parts

$$-F \int_{t-h}^{t} \mathrm{e}^{-a(s-t)} \dot{y}(s) \mathrm{d}s = -Fy(t) + F \mathrm{e}^{ah} y(t-h) - aF \int_{t-h}^{t} \mathrm{e}^{-a(s-t)} y(s) \mathrm{d}s$$

yielding

$$u(t-h) = -Fy(t) - 2aF \int_{-h}^{0} \mathrm{e}^{-as} y(t+s) \mathrm{d}s \tag{5}$$

Replacing the control law (5) in the nominal system (2):

$$\dot{y}(t) = a_0 y(t) + \int_{-h}^{0} \mathrm{d}(s) y(t+s) \,\mathrm{d}s$$

where

$$a_0 = (-a - bF), \quad \mathrm{d}(s) = -2abF \mathrm{e}^{-as}$$

When nonlinear disturbances are considered in the model, we obtain the following:

$$\dot{\bar{y}}(t) = a_0 \bar{y}(t) + \int_{-h}^{0} \mathrm{d}(s) \bar{y}(t+s) \,\mathrm{d}s + f(\bar{y}) \tag{6}$$

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where disturbance $f(\bar{y})$ satisfies

$$|f(\bar{y})| \leqslant \gamma |\bar{y}|, \quad \gamma > 0 \tag{7}$$

Remark 1
The system described by Equation (6) is not optimally controlled, but we need to find delay-dependent sufficient conditions, which guarantee the robust stability in closed loop. Because the model of Equation (1) is only an approximation, it makes sense to suppose the existence of non-modeled dynamics.

Sufficient conditions that assure robust stability under nonlinear disturbances satisfying (7) must be found. Consider the following Lyapunov Krasovskii functional

$$V(\bar{y}_t) = \alpha_0 \bar{y}^2(t) + \beta_0 \int_{t-h}^{t} \bar{y}^2(s)(s-t+h)\mathrm{d}s, \quad \alpha_0 > 0, \ \beta_0 > 0$$

This functional is unbounded radially because

$$\alpha_0 \|\bar{y}(t)\|^2 \leqslant V(\bar{y}_t) \leqslant (\alpha_0 + \beta_0 h^2) \|\bar{y}(t)\|_h^2$$

Now, we calculate the time derivative of $V(\bar{y}_t)$ along the trajectories of the system described by Equation (6):

$$\left.\frac{\mathrm{d}V(\bar{y}_t)}{\mathrm{d}t}\right|_{(6)} = (2\alpha_0 a_0 + \beta_0 h)\bar{y}^2(t) + 2\alpha_0 \bar{y}(t) \int_{-h}^{0} \mathrm{d}(s)\bar{y}(t+s)\mathrm{d}s + 2\alpha_0 \bar{y}(t) f(\bar{y}) - \beta_0 \int_{-h}^{0} \bar{y}^2(t+s)\,\mathrm{d}s$$

After some direct majorizations, and using (7):

$$\left.\frac{\mathrm{d}V(\bar{y}_t)}{\mathrm{d}t}\right|_{(6)} \leqslant -\int_{-h}^{0} [\bar{y}(t) \ \ \bar{y}(t+s)] \begin{bmatrix} 2\alpha_0 \dfrac{(a+bF-\gamma)}{h} - \beta_0 & 2\alpha_0 abF\mathrm{e}^{-as} \\ 2\alpha_0 abF\mathrm{e}^{-as} & \beta_0 \end{bmatrix} \begin{bmatrix} \bar{y}(t) \\ \bar{y}(t+s) \end{bmatrix} \mathrm{d}s$$

If matrix

$$M(s) = \begin{bmatrix} 2\alpha_0 \dfrac{(a+bF-\gamma)}{h} - \beta_0 & 2\alpha_0 abF\mathrm{e}^{-as} \\ 2\alpha_0 abF\mathrm{e}^{-as} & \beta_0 \end{bmatrix} > 0$$

then we conclude that the derivative along the trajectories of system (6) is stable. Matrix $M(s)$ is positive definite if

$$0 < 2\alpha_0 \frac{(a+bF-\gamma)}{h} - \beta_0 \quad \text{for all } t \geqslant 0$$

and

$$0 < [2Fb\alpha_0\beta_0 + 2a\alpha_0\beta_0 - h\beta_0^2 - 2\gamma\alpha_0\beta_0 - 4F^2a^2b^2h\alpha_0^2\mathrm{e}^{-2as}] \quad \text{for } s \in [-h, 0]$$

Observe that $-F^2a^2b^2h\alpha_0^2\mathrm{e}^{-2as} \leqslant -F^2a^2b^2h\alpha_0^2$, because $s \in [-h, 0]$. Consequently,

$$\begin{aligned} 0 &< [2Fb\alpha_0\beta_0 + 2a\alpha_0\beta_0 - h\beta_0^2 - 2\gamma\alpha_0\beta_0 - 4F^2a^2b^2h\alpha_0^2\mathrm{e}^{-2as}] \\ &< 2Fb\alpha_0\beta_0 + 2a\alpha_0\beta_0 - h\beta_0^2 - 2\gamma\alpha_0\beta_0 - 4F^2a^2b^2h\alpha_0^2 \end{aligned}$$

The following lemma is established:

Lemma 2
Consider the disturbed system (6) and (7). System of Equation (6) is robustly stable under nonlinear disturbances if positive constants α_0 and β_0 exist such that the following inequalities are satisfied:

$$\begin{aligned} 0 &< 2\alpha_0 \frac{(a+bF-\gamma)}{h} - \beta_0 \\ 0 &< 2Fb\alpha_0\beta_0 + 2a\alpha_0\beta_0 - h\beta_0^2 - 2\gamma\alpha_0\beta_0 - 4F^2a^2b^2h\alpha_0^2 \end{aligned} \tag{8}$$

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3. EXPERIMENTAL RESULTS

In this section we present experimental results for a temperature control system. The system is a box that has three fans actuated by three DC motors (3–12 V DC), a source of heat (an electrical grid of 120 V AC, but we introduced 17.5 V AC), a temperature sensor (integrated circuit LM35), and a tunnel as output in the box. We take as input the three fans whereas the feeding source of heat remains constant. Figure 1 shows the instrumentation diagram of the system:

The approximated linear model of the plant is obtained by the open loop Ziegler–Nichols method [28]:

$$\frac{Y(s)}{U(s)} = G(s) = \frac{0.1\mathrm{e}^{-14s}}{226s+1} \tag{9}$$

Equation (9) clearly complies with model of Equation (1), where $Y(s)$ is the mapped temperature and $U(s)$ is the voltage applied to the fans. This approximation was obtained by analyzing the step response of the system (Figure 2).

The external temperature was 25°C. In the time domain we have that

$$\dot{y}(t) = -0.0044248y(t) + 0.00044248u(t-14) \tag{10}$$

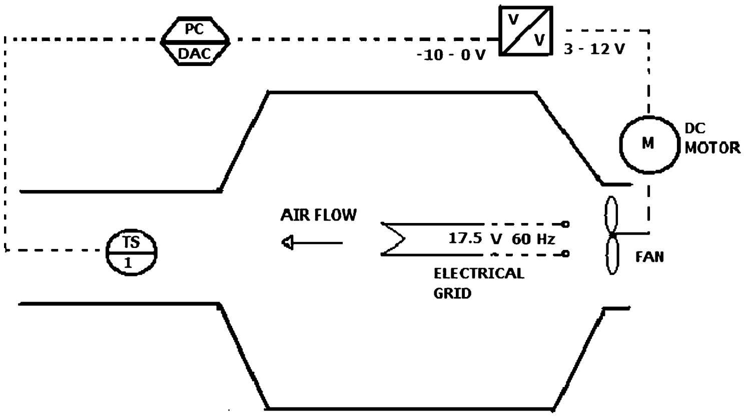

Figure 1. Instrumentation diagram.

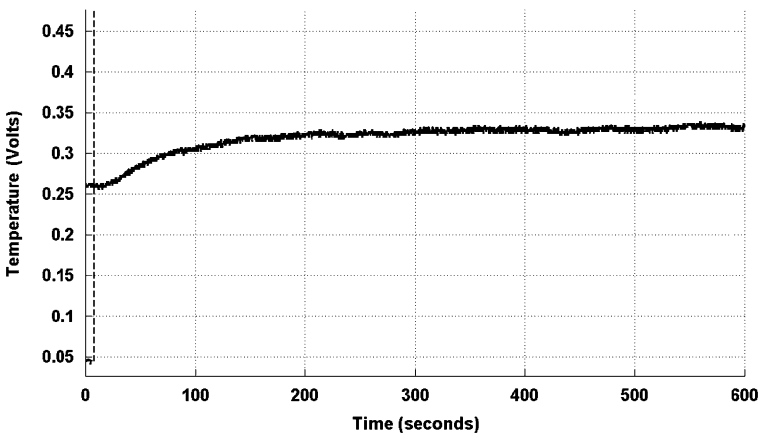

Figure 2. Step response of the process.

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here the units are in volts (100 mV represent 1°C) and the time is expressed in seconds. Using the equivalent controller given by (5) the optimal control is

$$u(t-14) = -F\mathrm{e}^{-0.0044248} y(t) - 0.0088496F \int_{-14}^{0} \mathrm{e}^{-0.0044248s} y(t+s)\,\mathrm{d}s$$

where $F = 70$. This value was obtained by the solution of the Riccati equation and $Q = 1260$ and $R = 0.02$. The closed loop is

$$\dot{y}(t) = a_0(t)y(t) + \int_{-h}^{0} \mathrm{d}(t+s)y(t+s)\mathrm{d}s$$

where

$$a_0 = -0.0073468$$

$$\mathrm{d}(s) = -0.0002741\mathrm{e}^{-0.0044248s}$$

By employing the conditions given by Lemma (2), we obtain the following inequalities

$$0 < g_1 = 2\alpha_0 \frac{(0.035398-\gamma)}{14} - \beta_0$$

$$0 < g_2 = 0.070797\alpha_0\beta_0 - 14\beta_0^2 - 1.0519 \times 10^{-6}\alpha_0^2 - 2\gamma\alpha_0\beta_0$$

Choosing $\alpha_0 = 1000$, $\beta_0 = 0.1$, $\gamma = 0.001$, the former inequalities are satisfied.

$$0 < g_1 = 4.814, \quad 0 < g_2 = 5.6878$$

Hence, the mathematical analysis demonstrates that the system is robustly stable under nonlinear disturbances. Our aim is to implement and validate the control law in real conditions.

3.1. Implementation of the control law

We need to control the temperature in the box considering the fans as actuators. By employing the optimal controller (4), we have rewritten the control law as:

$$u(t-h) = -F\mathrm{e}^{ah} y(t-h) - F\int_{-h}^{0} \mathrm{e}^{-as} bu(t+s-h)\mathrm{d}s$$

If we approximate this control law by a quadrature method (composed Simpson's rule), we obtain the following controller:

$$u(t-h) \simeq -F\mathrm{e}^{ah} y(t-h) - F\Bigg[\frac{g}{3}(\mathrm{e}^{ah} bu(t-2h) + bu(t-h)) + \frac{2g}{3}\sum_{k=1}^{q-1}(\mathrm{e}^{-ax_{2k}} bu(t+s_{2k}-h)) + \frac{4g}{3}\sum_{k=1}^{q}(\mathrm{e}^{-ax_{2k-1}} bu(t+s_{2k-1}-h))\Bigg] - \mathrm{adj}$$

where $g = h/2q$, q is the number of partitions, $s_k = y_k = -h + (h/2q)k$. We performed a change of variable in order to reach a Set Point (SP) different than zero, as we want to cool the box. This change of variable is $\bar{y}(t-h) = y(t-h) - \mathrm{SP}$. We add a constant term ($\mathrm{adj} = \mathrm{SP}/(T)(b)$) in the control law that defines the approximated value in the actuators to hold the temperature when the error is zero.

As the process has a large constant time (226 s), and a delay of 14 s, we consider that the digital implementation is almost continuous with a sample period of 1 s. The control is implemented

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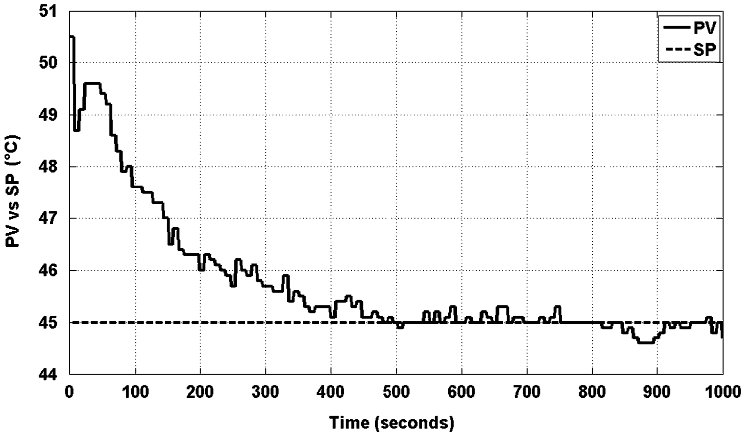

Figure 3. PV versus SP.

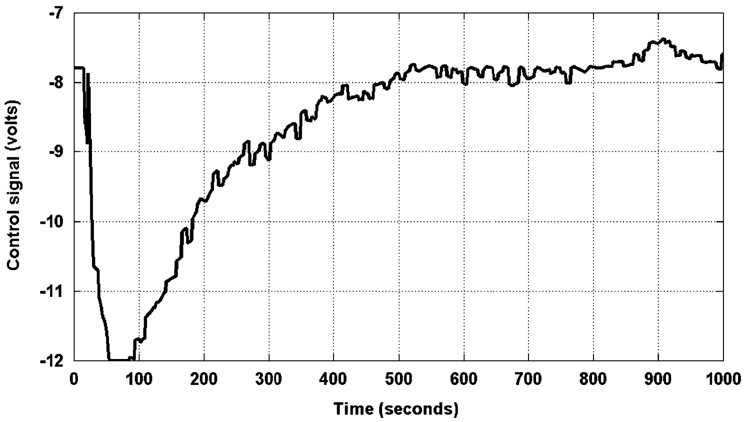

Figure 4. Optimal control signal.

on a personal computer with an Intel Pentium 4 processor, 2 GHz, 1 GB RAM, and a National Instruments data acquisition card PCI-6071 E. The software is Lab View of National Instruments, V. 7.1. The initial temperature in the camera was 50°C, and the reference was set at 45°C. The external temperature at the initial moment was 25°C. A sample compression block in Lab View was used in order to filter the noise of the sensor. Figure 3 illustrates the obtained results of the process variable (PV).

Figure 4 shows the control signal.

Remark 3
When temperature reaches or is around the SP, some problems appear in order to maintain the temperature in the SP, because the value *adj* depends on values that are not precise but approximated (T, b). The particular choosing of the parameters Q and R penalizes the convergence (Q) and the control law (R) can take a large value, but another choosing of Q and R could give different results.

Now, consider the case when temperature is risen. We elevated the applied voltage in the grid to 20 V. This situation represents a disturbance in the plant. In this experiment the external temperature was 25°C, the initial temperature in the box was 50°C, and the SP was fixed at 45°C. The disturbance was introduced in the second 1500. The following illustration shows the obtained results (Figure 5).

The control signal is depicted in Figure 6.

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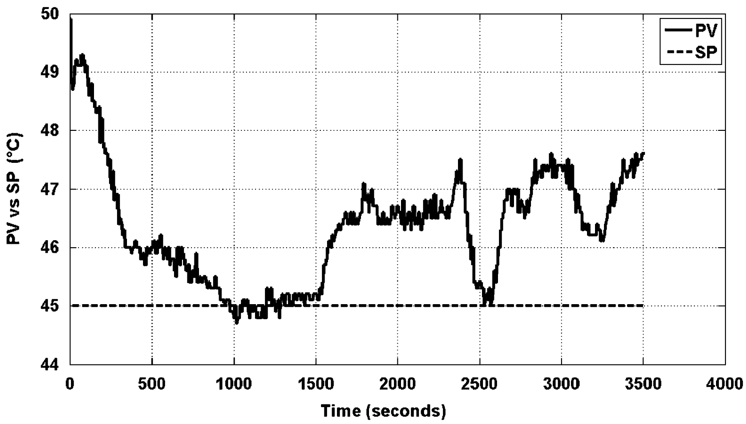

Figure 5. PV versus SP under disturbances.

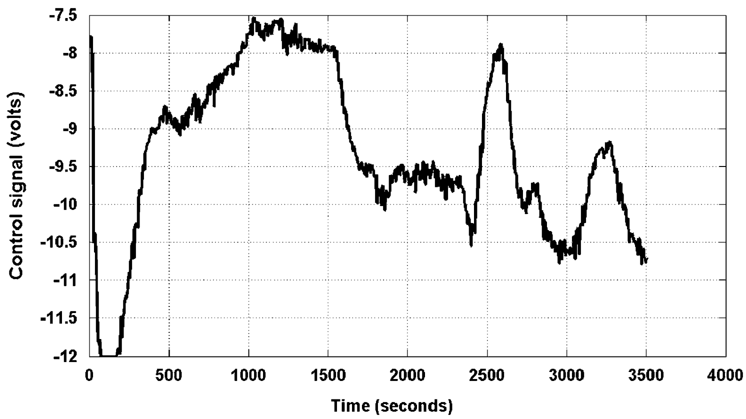

Figure 6. Control signal with disturbance in the plant.

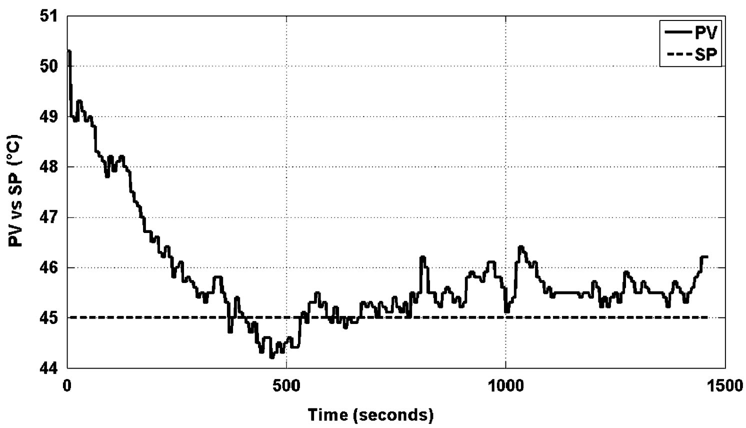

Figure 7. PV versus SP under disturbances when the gain is increased.

We observe that the system remains stable, but the performance becomes poor. An alternative is to elevate de controller gain, so we choose $F=100$ and introduce a disturbance in the second 700. Figure 7 shows the obtained results.

The control signal is shown in Figure 8.

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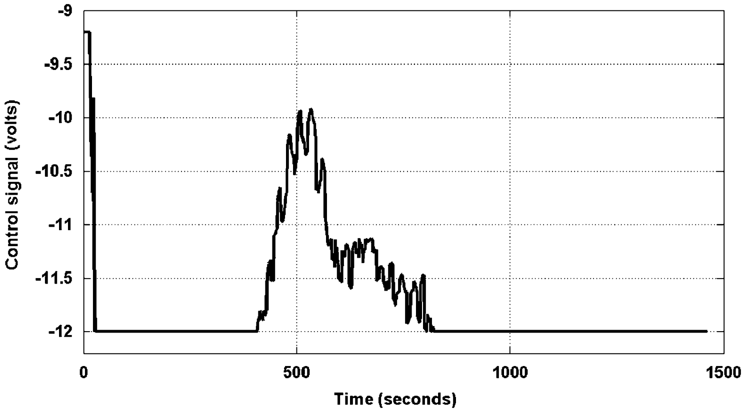

Figure 8. Control signal with disturbance in the plant.

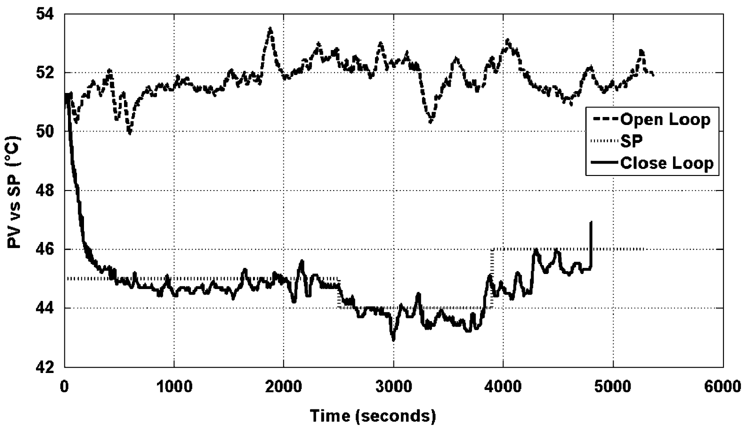

Figure 9. PV in open and closed loop.

Remark 4
In Figure 7, we observe that if the gain ought to be elevated, the system's response displays a better performance and the disturbances are not affected too much, but in Figure 8 the control signal remains saturated for longer.

3.2. Response to trajectory tracking

In this section we present a more demanding task for the optimal controller. It has to adjust or adapt as different SPs where set on line. This is known as trajectory tracking. In order to compare the response in open and closed loop, we made an experiment applying the voltage corresponding to the temperature of the SP ($\mathrm{adj} = \mathrm{SP}/(T)(b)$). Clearly, if the initial conditions are different than the SP, the plant in open loop does not track the reference. Figure 9 shows the obtained results in both open and closed loop, applying optimal the control law.

Figure 10 shows the control signal and the voltage applied in open loop.

A good performance of the plant can be observed and the control signal stays saturated briefly. In the next section we present a comparison with a PID controller. For this experiment a composed Simpson's rule was employed, with different step numbers. Figure 11 shows the responses using different step numbers.

Figure 12 shows the optimal control law signal using three different values for the step.

Observe that the behavior of the plant in closed loop is similar. However, with $q = 100$ the plant displays smoother trajectory and the control 'reacts' faster when a change in the reference occurs.

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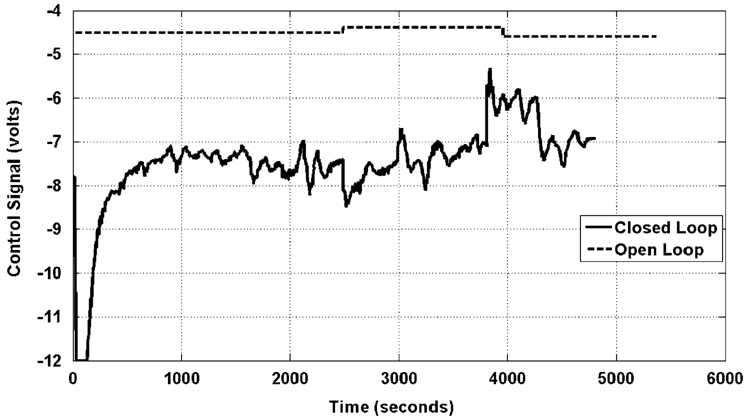

Figure 10. Control signals in open and closed loop.

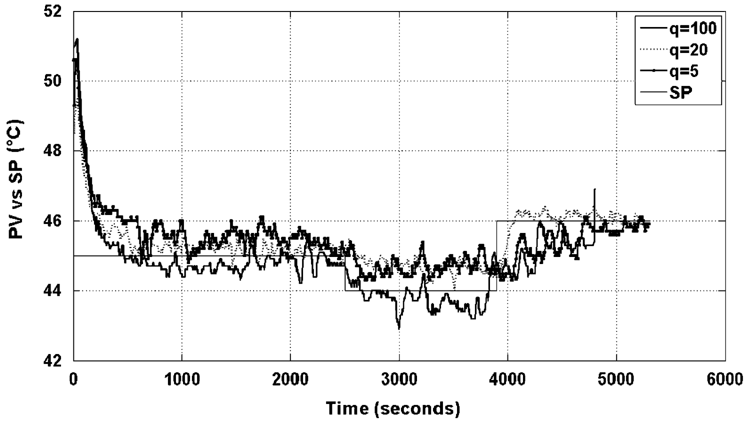

Figure 11. Comparison of PV responses with optimal control law approximated by composed Simpson's rule.

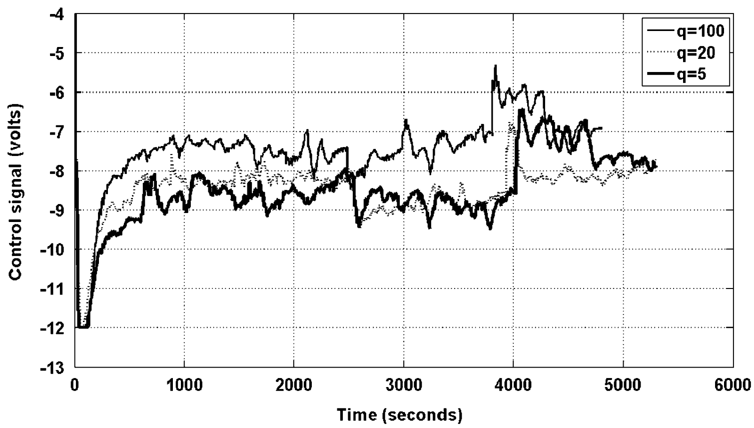

Figure 12. Implemented optimal control signals with the composed Simpson's rule.

4. COMPARISON WITH AN INDUSTRIAL PID CONTROLLER

In order to compare the performance of the proposed optimal controller, we carried a series of experiments with an industrial PID controller (Honeywell DC1040). This controller reads signals

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coming from a thermocouple, element that gives finer readings that the low-cost sensor we employed for our optimal controller. The Honeywell DC1040 has the industrial standard output signal of 4–20 mA; it also possesses dead-band time compensation, which is a type of time-delay compensation and the controller autotunes. This type of controller is widely used in the industry due to facilities of programming and accuracy. Experiments were conducted with the industrial PID controller using the same plant where the optimal control law was tested. The results are shown in the following subsections.

4.1. Robust PID

The performance of the PID controller is improved by using different tuning techniques. We selected the D-partitions method in order to obtain a robust PID controller. We calculated the following stability zones for the PID controller when the approximated model (9) is considered: $K_p \in [-10, 259.9]$. By choosing $K_p = 30$ (BP = 3.3), the stability region for K_i is $[0, 2.36]$, $K_i = K_p/T_i$. The value of $K_i = 0.7$ ($T_i = 42.85$ s); with which we calculated the stability region for K_d, $K_d \in [0, 769.5]$, $K_d = K_p T_d$. We choose $K_d = 150$ ($T_d = 5$ s). Observe that the interval for K_i and K_d depends on the selected values of K_p and K_i respectively, see [3]. The response of the plant is illustrated in Figure 13.

Figure 14 shows the control signals of the optimal control and PID controller.

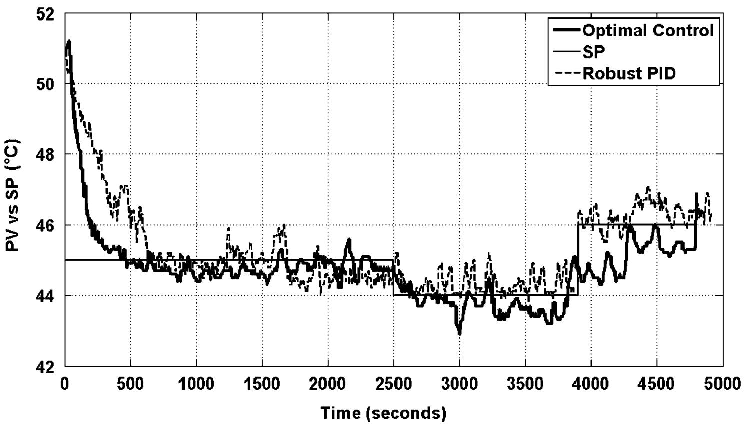

Figure 13. PV versus SP with optimal control and robust PID.

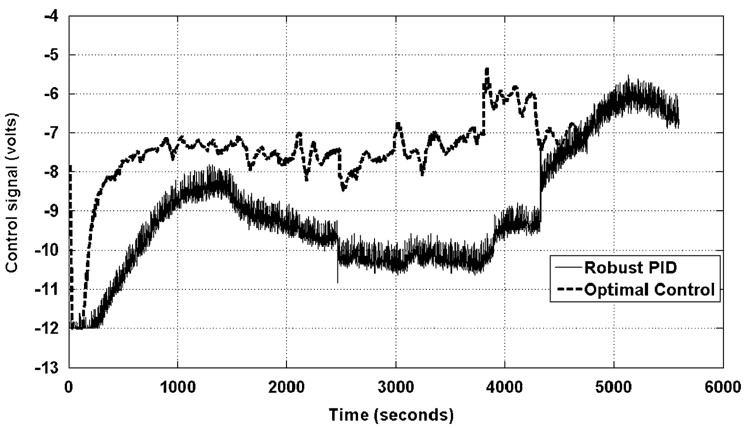

Figure 14. Optimal control signal versus robust PID signal.

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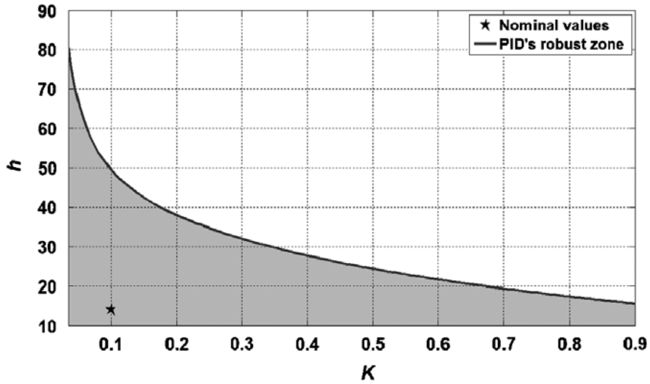

Figure 15. Robust stability zone for the PID controller.

Table I. Values of the ISE for different experiments.

Controller	Value of the ISE
Optimal control	0.35202450
PID (robustly tuning)	0.32967950

Figure 15 shows the robust stability zone for the PID controller.

In Table I we summarize the responses using the integral square errors of the previous control strategies.

Our results prove that the optimal control law implemented displays a similar performance, in terms of the Integral Square Error (ISE), than the industrial PID. Nevertheless, we must outline that the best performance of the PID was achieved when it was robustly tuned. Our experiments indicate that the optimal controller is more energy efficient, since it does not stay in the saturation region as much as the PID does.

4.2. *Composed trapezoidal rule*

In this section the integral term in the optimal control law is approximated by the composed trapezoidal rule with different step numbers. The controller could be calculated as:

$$u(t-h) \simeq -F\mathrm{e}^{ah}\bar{y}(t-h) - F\left[\frac{g}{3}(\mathrm{e}^{ah}bu(t-2h)+bu(t-h))+g\sum_{k=1}^{q-1}(\mathrm{e}^{-ax_k}bu(t+s_k-h))\right] - \mathrm{adj} \quad (11)$$

where $g = h/q$, q is the number of partitions, $s_k = x_k = -h + (h/q)k$. A change of variable in order to reach a SP different than zero is done. In fact, as we want to cool the box this change of variable is $\bar{y}(t-h) = y(t-h) - \mathrm{SP}$. The constant term $(\mathrm{adj} = \mathrm{SP}/(T)(b))$ is added in the control law which defines the approximated value in the actuators to hold the temperature when the error is zero. Figure 16 shows the responses of the plant in closed loop with the control law (11) with three values for the integration step q.

Figure 17 shows the control signal applied to the actuators.

The following table shows comparative results using the performance index given by (3) for the control law implemented with the Composed Simpson and trapezoidal rules.

We observe a decrement in the numerical value of the index when the number of partitions are increasing (Table II).

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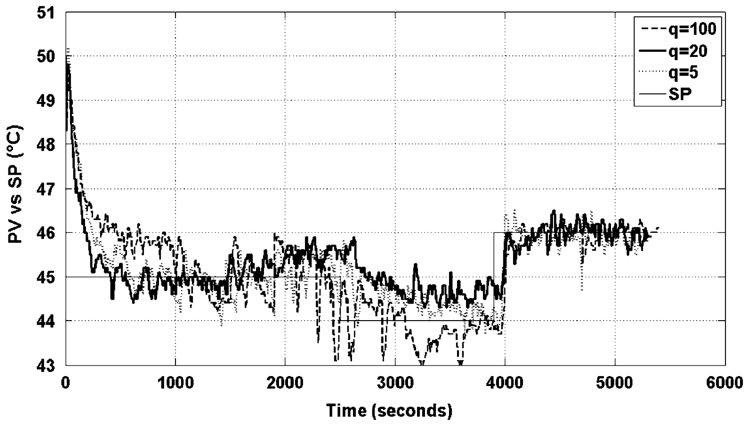

Figure 16. Comparison of PV responses with optimal control law approximated by composed Trapezoidal rule.

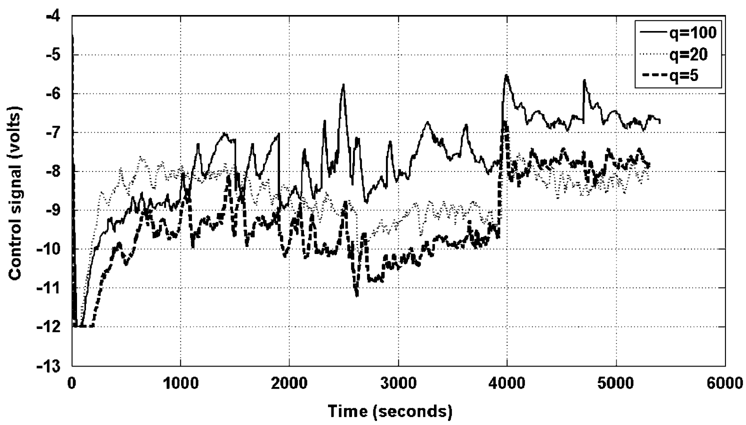

Figure 17. Implemented optimal control signals with the composed Trapezoidal rule.

Table II. Values of the performance index.

Partitions number (q)	Composed Simpson's rule	Composed Trapezoidal rule
100	8.8814×10^5	8.99187×10^5
20	9.08941×10^5	9.05822×10^5
5	9.17901×10^5	9.04839×10^5

5. CONCLUSIONS

We obtained satisfactory experimental results in the implementation of the optimal control for a type of system with time delay in the input. By Lyapunov Krasovskii analysis, we provided sufficient conditions to satisfying robust stability for nonlinear disturbances. These experiments give evidence about feasibility of the approximated controller involving distributed time delays. With this report we fill a gap in the scientific literature for this class of systems by actually controlling a real plant and providing significant experimental results. The comparison with and industrial PID controller demonstrates both, the more efficient response of the optimal control, and the complexity inherent to control this type of systems. Future work includes experimenting with numerical optimization in order to tune the PID controller, and implementing nonlinear optimal controllers for time delay systems.

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