Artificial Neural Networks

for people in a hurry, in at most four hours!

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Topics

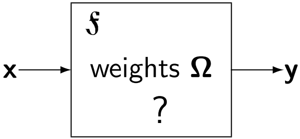
1. Artificial Neural Networks
   - Architectures
   - Learning paradigms
2. Feedforward Artificial Neural Networks
   - Multilayer perceptron
   - Radial Basis Function
3. Associative Artificial Neural Networks
   - Hebb's rule
   - Characteristics of the conditioning process
4. Neurofuzzy Systems
   - Basic extensions of crisp neurons
5. Support Vector Machines
6. Spiking Neurons

Topics

Basic definitions

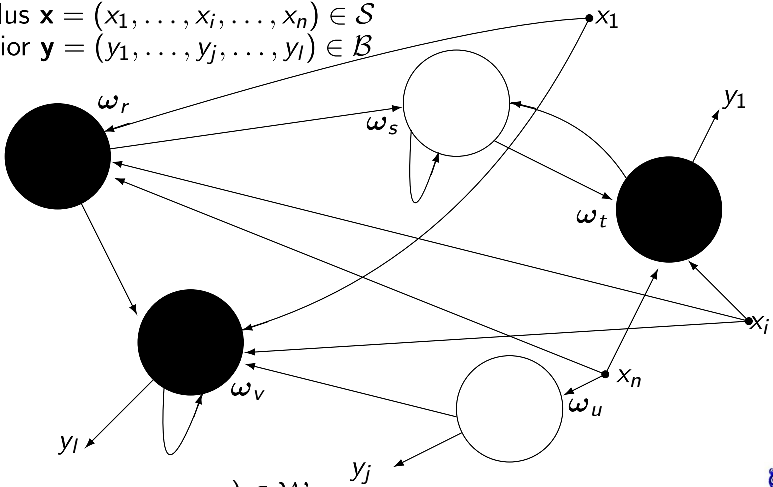
Definition

An artificial neural network is a function $\mathfrak{f}\colon \mathcal{S} \times \mathcal{W} \to \mathcal{B}$, where $\mathcal{S}$ represents the stimuli space, $\mathcal{B}$ the behaviors space and $\mathbf{y} = \mathfrak{f}(\mathbf{x}; \boldsymbol{\Omega})$ is the rule of correspondence, with $\mathbf{x} \in \mathcal{S}$, $\mathbf{y} \in \mathcal{B}$ and $\boldsymbol{\Omega} \in \mathcal{W}$ the weight parameters to be determined through a proper training or learning process inside the weight space $\mathcal{W}$.



General structure

Stimulus $\mathbf{x} = (x_1, \ldots, x_i, \ldots, x_n) \in \mathcal{S}$
Behavior $\mathbf{y} = (y_1, \ldots, y_j, \ldots, y_l) \in \mathcal{B}$



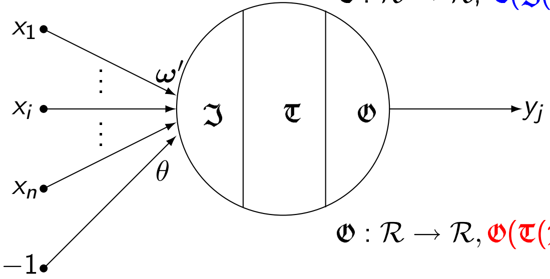
$\mathbf{\Omega} = (\ldots, \omega_r, \ldots, \omega_v, \ldots) \in \mathcal{W}$

General neuron structure

Stimulus $\mathbf{x} = (x_1, \ldots, x_i, \ldots, x_m) \in \mathcal{S}'$

$\mathfrak{I} : \mathcal{S}' \times \mathcal{W}' \to \mathcal{R}$, $\mathfrak{I}(\mathbf{x}; \omega', \theta) \in \mathcal{R}$

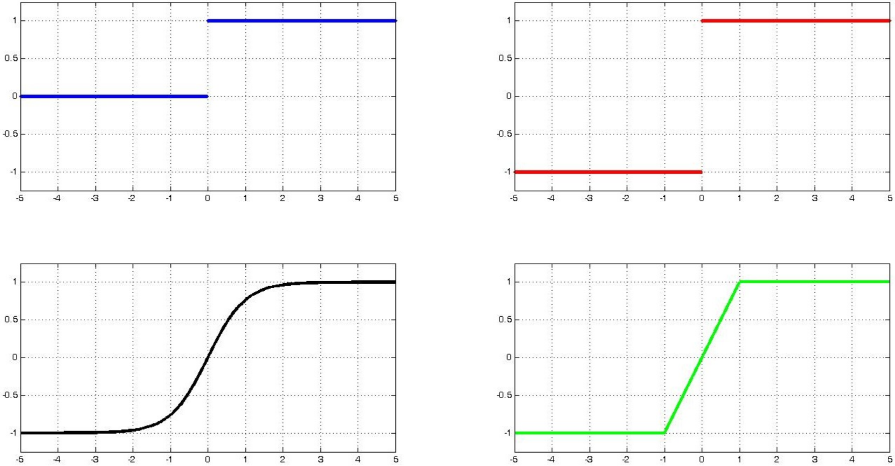
$\mathfrak{T} : \mathcal{R} \to \mathcal{R}$, $\mathfrak{T}(\mathfrak{I}(\mathbf{x}; \omega', \theta)) \in \mathcal{R}$



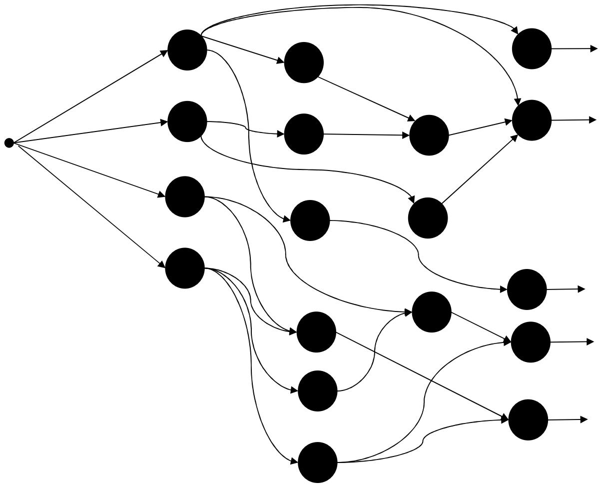
$\mathfrak{O} : \mathcal{R} \to \mathcal{R}, \mathfrak{O}(\mathfrak{T}(\mathfrak{I}(\mathbf{x}; \omega', \theta))) \in \mathcal{R}$

Transfer or activation functions

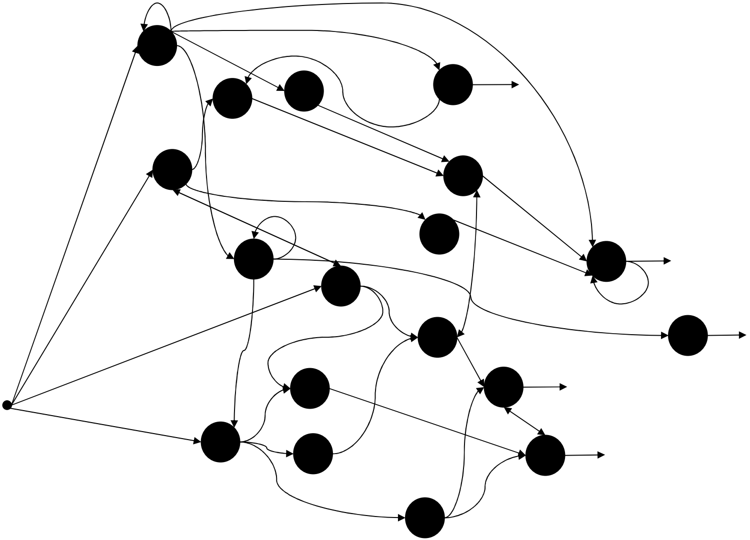
(Chapter 2 and 11 (Network function) MatLab nnd command)



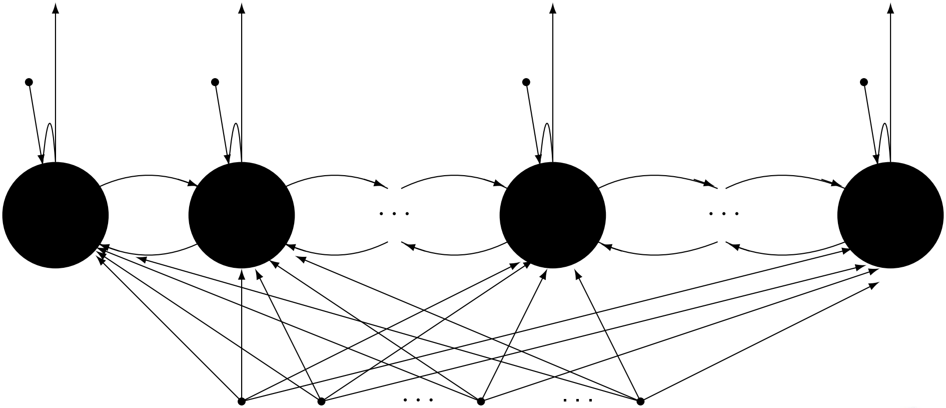
Feedforward artificial neural networks



Feedback artificial neural networks



Competitive artificial neural networks



Supervised learning

Main characteristics

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4. Square error provides theoretical basis for learning procedures.

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1. The most difficult way of learning.
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4. Looking for correlation between weights and stimuli.
5. Hebb's rule provides theoretical basis for learning procedures.

Topics

Simple perceptron

Main properties (Chapters 3 and 4 MatLab nnd command)

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4. Classification problems with two or more classes can be solved without difficulty only if they are linearly separable.
5. Impossible to solve the XOR problem (non linearly separable problem), without introducing an additional layer of neurons.

Multilayer perceptron

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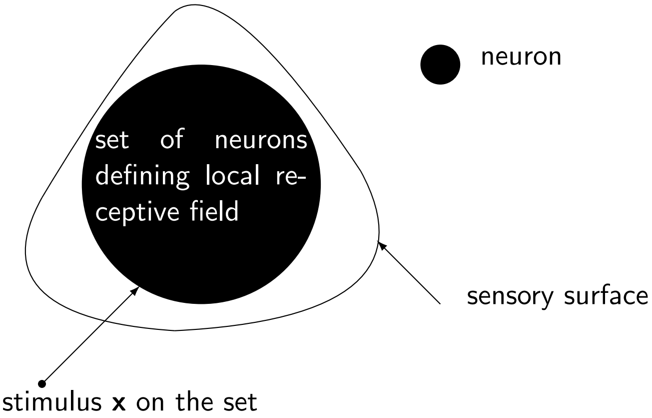
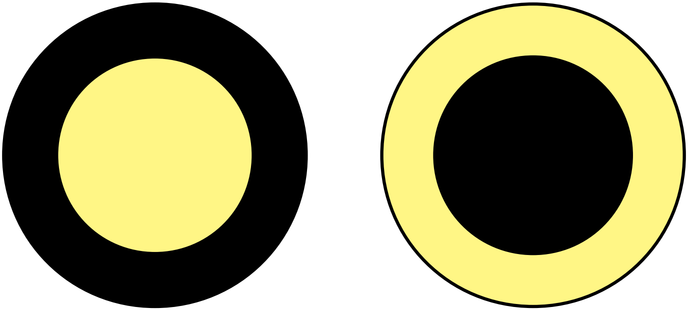
$$\omega_{new} = \omega_{old} - \eta \frac{\partial \mathfrak{E}^2}{\partial \omega},$$

where $\mathfrak{E}^2$ is the square total error as a function of the weights ω and $\frac{\partial}{\partial \omega}$ is the gradient operator.

Local receptive field



Types of local receptive fields



on center - off surround

off center - on surround

Mathematical model

Fourier's transform

$f : \mathcal{R} \rightarrow \mathcal{R}$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} g(y) e^{ixy} dy,$$

where g is the inverse Fourier's transform

Inverse Fourier's transform

$$g(y) = \int_{-\infty}^{+\infty} f(z) e^{-izy} dz$$

Dirac's delta

Dirac's delta definition

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(z) \int_{-\infty}^{+\infty} e^{i(x-z)y} dydz = \int_{-\infty}^{+\infty} f(z)\delta(x-z)dz$$

$$\delta(x-z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i(x-z)y} dy$$

Dirac's delta properties

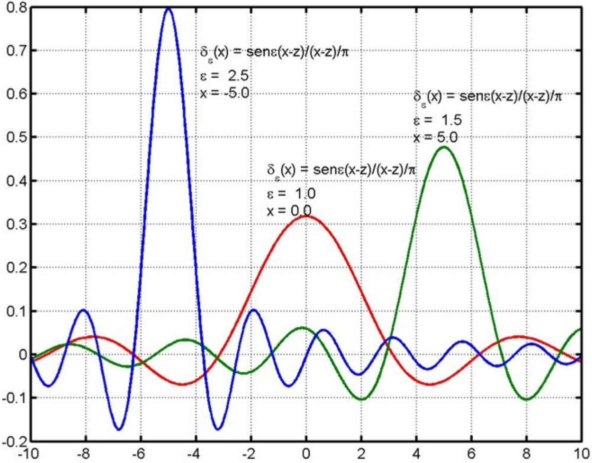
1. $$\int_{-\infty}^{\infty} \delta(x-z)dz = 1$$

2. $$\delta(x-z) = \lim_{\epsilon \to +\infty} \frac{\sin \epsilon(x-z)}{\pi(x-z)}$$

3. Dirac's delta is a sequence of functions
$$\delta_{\epsilon}(x-z) = \frac{\sin \epsilon(x-z)}{\pi(x-z)}, 0 < \epsilon$$

4. Dirac's delta es an even function
$$\delta(x-z) = \delta(z-x)$$

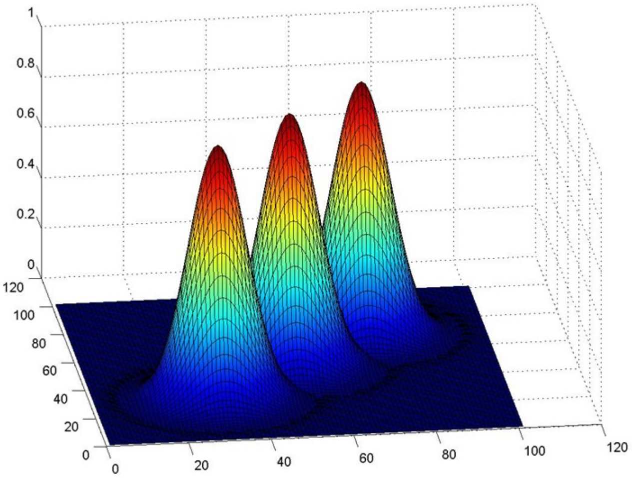
Some sequence examples



Some approximations to Dirac's delta sequence

1. For z's very close to x, $\delta_\epsilon(x - z) \approx \frac{\epsilon}{\pi}, 0 < \epsilon$
2. $e^{-\lambda(x-z)^2}, 0 < \lambda$
3. $\frac{1}{(x-z)^2+c^2}$
4. Two dimensional Dirac's delta approximation $e^{-\lambda_1(x-z_1)^2-\lambda_2(y-z_2)^2}, 0 < \lambda_1, \lambda_2$
5. Another two dimensional Dirac's approximation $\frac{\sin \epsilon_1(x-z_1) \sin \epsilon_2(y-z_2)}{\pi^2(x-z_1)(y-z_2)}$

Graphics of some two dimensional sequences



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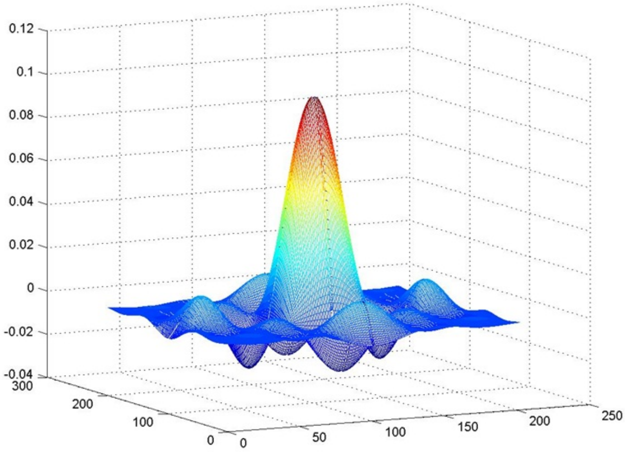

Summary (first steps to build rbf artificial neural network)

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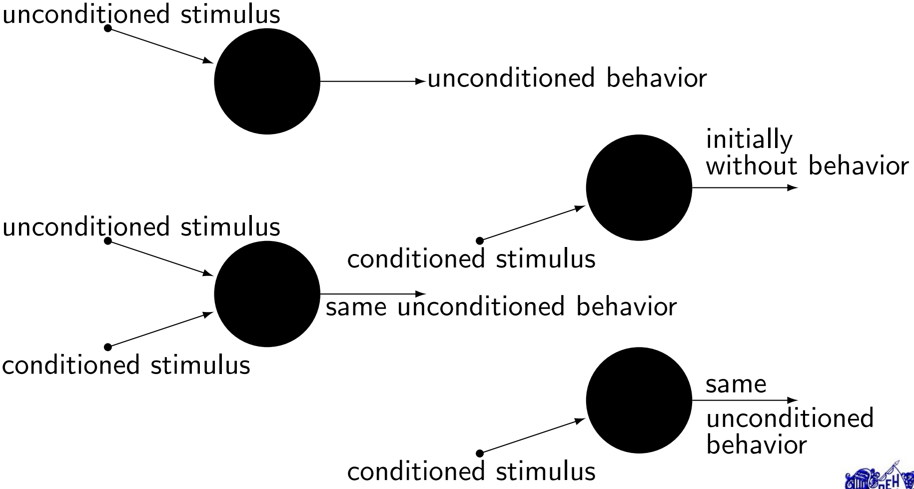
8. First layer with parameters of basis functions as weights to be determined through training process (supervised learning).
9. Second layer with single neuron with weights to be determined through training process (supervised learning).

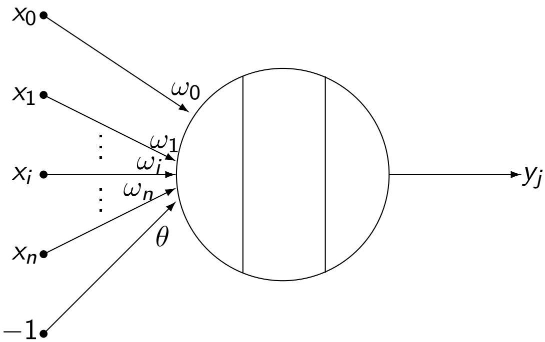
Topics

Pavlov's conditioning



Conditioning neuron structure

Stimulus $\mathbf{x} = (x_0, x_1, \ldots, x_i, \ldots, x_n, -1)$
Weights $\boldsymbol{\omega} = (\omega_0, \omega_1, \ldots, \omega_i, \ldots, \omega_n, \theta)$



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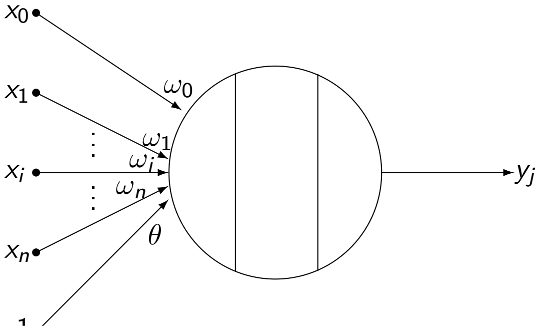

Observation!

Since the unconditioned stimulus is previously learnt, the weights ω_0 and θ are not modified along the learning process. The conditioned stimulus needs to be learnt, so the corresponding weights need to be modified.

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Main properties (Chapter 14 (Competitive learning))

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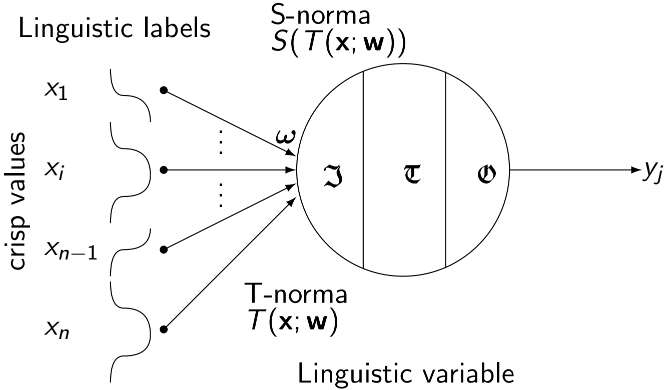
7. The weights are bounded, as well.

Topics

Focus of the main thrust of fuzzy neural nets

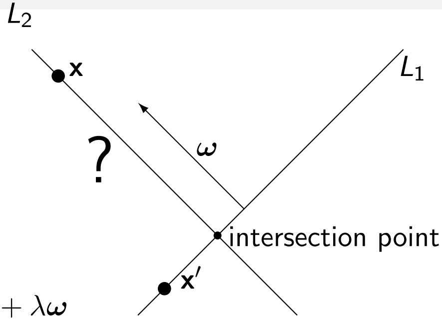
1. The fuzzification of the dendritic inputs.
2. The aggregation operation of a conventional neuron.

Basic structure of a fuzzy neuron



Topics

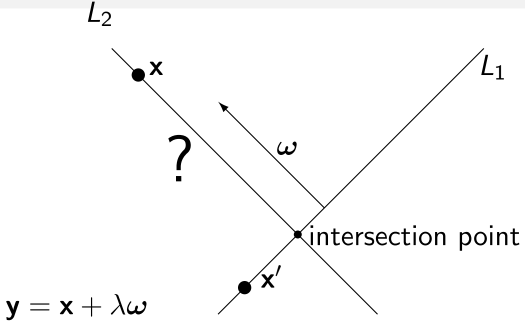
Hard margin support vector machine



$$\mathbf{y} = \mathbf{x} + \lambda\omega$$

$$\mathbf{y} = \mathbf{x}' + \lambda'\omega^{\perp}$$

Hard margin support vector machine



Line equation for L_2 and L_2

line equation L_2

$$\mathbf{y} = \mathbf{x} + \lambda\boldsymbol{\omega}, \lambda \in \mathcal{R}$$

line equation L_1

$$\mathbf{y} = \mathbf{x}' + \lambda'\boldsymbol{\omega}^{\perp}, \lambda' \in \mathcal{R}$$

Shortest distance from $\mathbf{x}$ to line L_1

In the intersection point there do exist scalars λ and λ' such that the following holds

$$\mathbf{x} + \lambda\boldsymbol{\omega} = \mathbf{x}' + \lambda'\boldsymbol{\omega}^{\perp}$$

After multiplying with dot product both sides of the equation by $\boldsymbol{\omega}^{\perp}$

$$\boldsymbol{\omega}^{\perp} \cdot \mathbf{x} = \boldsymbol{\omega}^{\perp} \cdot \mathbf{x}' + \lambda'||\boldsymbol{\omega}||^2$$

So that, the scalar λ' in the intersection point is given by the equation

$$\lambda' = \frac{\boldsymbol{\omega}^{\perp} \cdot (\mathbf{x} - \mathbf{x}')}{||\boldsymbol{\omega}||^2}$$

Therefore, the shortest distance from $\mathbf{x}$ to line L_1 is given by the expression

$$\frac{|\boldsymbol{\omega}^{\perp} \cdot (\mathbf{x} - \mathbf{x}')|}{||\boldsymbol{\omega}||}$$

Shortest distance

This is so, since

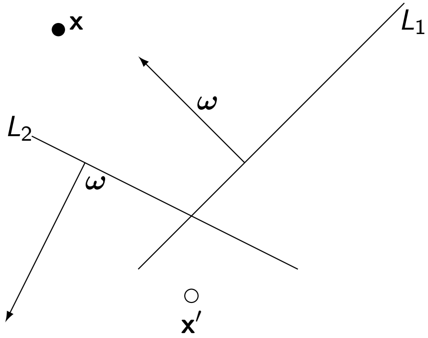
$$\frac{|\omega^{\perp} \cdot (\mathbf{x} - \mathbf{x}')|}{||\omega||} = \frac{\omega^{\perp} \cdot \mathbf{x} - \omega^{\perp} \cdot \mathbf{x}'}{||\omega||}$$

And, after adding a zero value

$$\frac{\omega^{\perp} \cdot \mathbf{x} + b - \omega^{\perp} \cdot \mathbf{x}' - b}{||\omega||} = \frac{\omega^{\perp} \cdot \mathbf{x} + b}{||\omega||} = \frac{c}{||\omega||}$$

($\omega^{\perp} \cdot \mathbf{x}' - b = 0$ is the equation of L_1 and $\omega^{\perp} \cdot \mathbf{x} + b = c$ is the equation of the line passing through $\mathbf{x}$ and parallel to L_1)

Two classes case



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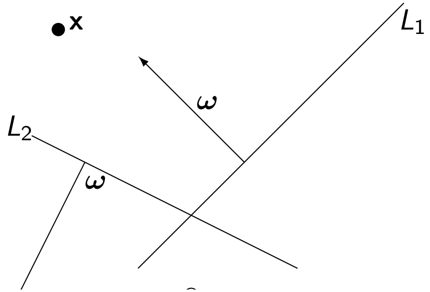

Important question

What's the minimum weight vector ω maximizing the distance

$$\frac{c}{||\omega||}?$$

Hard margin support vector machine

Problem

Minimize

$$Q(\omega) = \frac{1}{2}||\omega||^2$$

Subject to the constraints

$$y_i(\boldsymbol{\omega}^{\perp}\mathbf{x}_i + b) \geq 1, \forall i = 1, 2, \ldots, M$$

Data x_i's that satisfy the equalities are called support vectors (deleting all the data that satisfy the strict inequalities do not affect the resulting hyperplane)

Topics